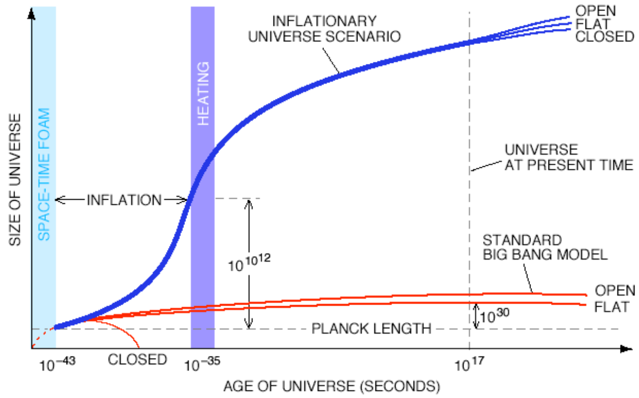
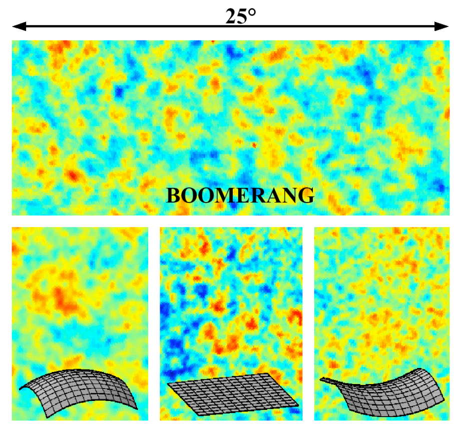
A Bayesian View of the Universe: From Massive Data Sets To Physical Understanding

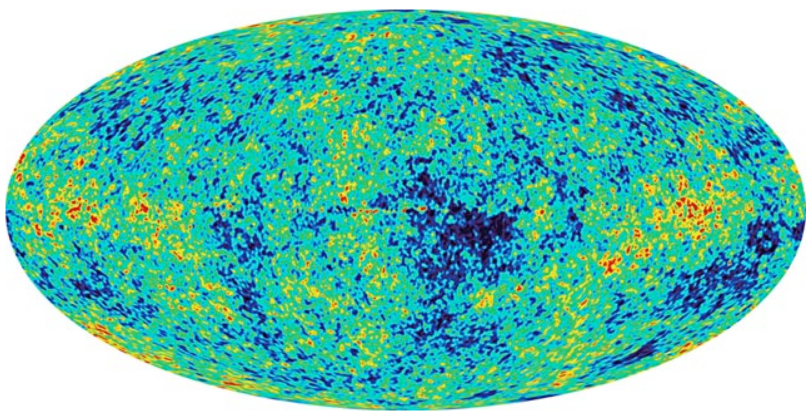
Introduction

Overview of Cosmology



Bayesian Inference





Current and Future Data

The Computational Challenge!

From the Proceedings of the National Academy of Sciences
Volume 15 : March 15, 1929 : Number 3

A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY AMONG EXTRA-GALACTIC NEBULAE

By Edwin Hubble
Mount Wilson Observatory, Carnegie Institution of Washington

Communicated January 17, 1929

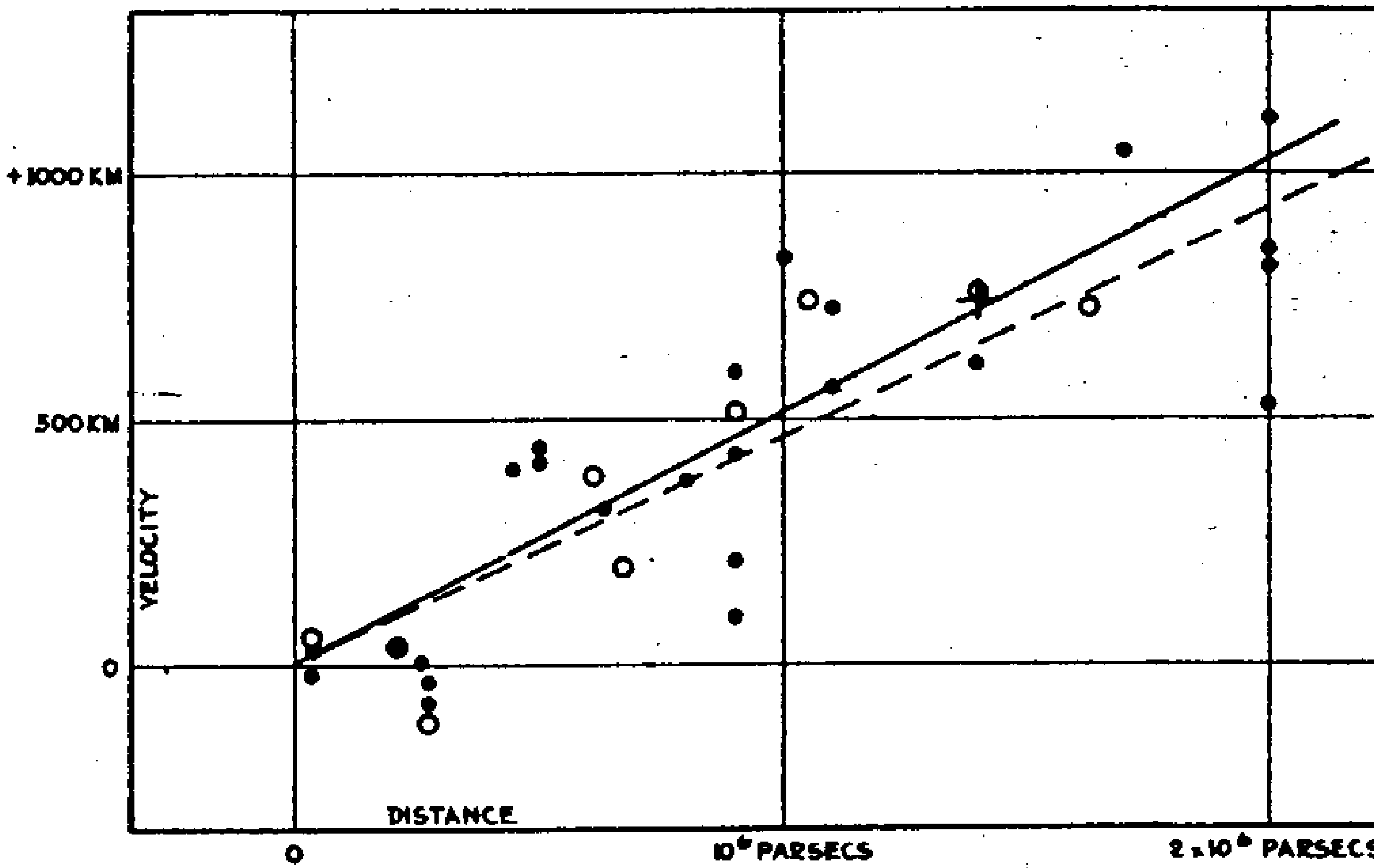


FIGURE 1

Inferring the Expansion Rate (and Age) of the Universe

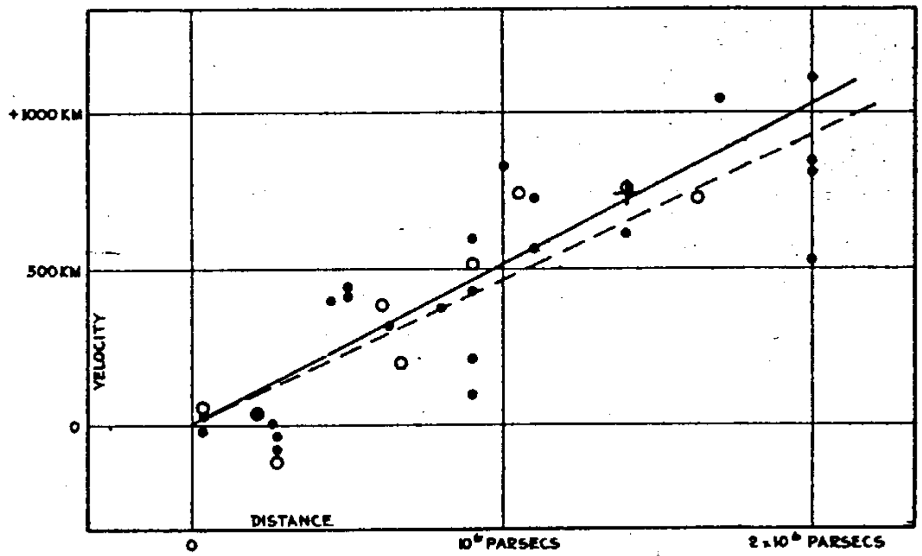
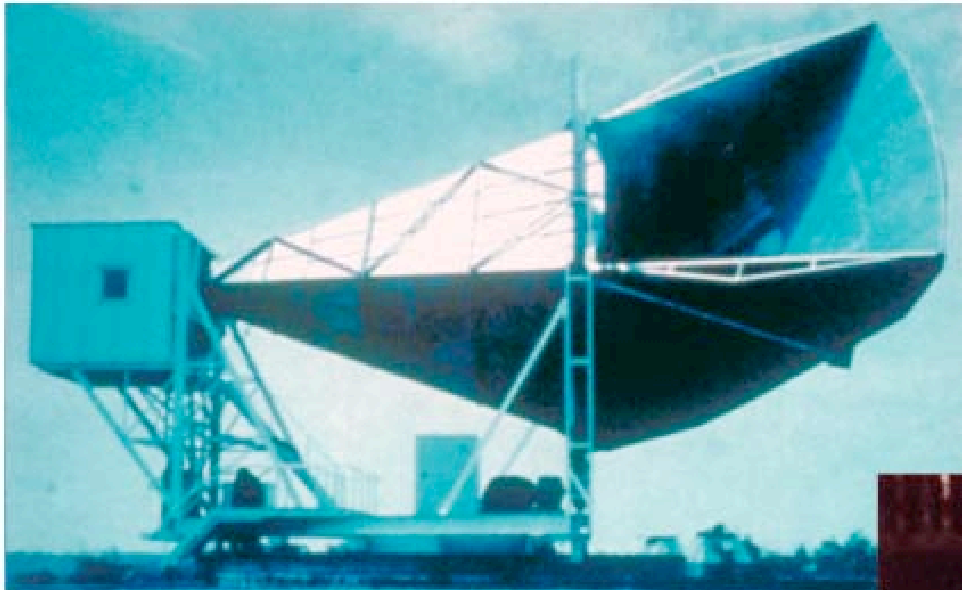


FIGURE 1

$$v_i = H_0 r_i$$

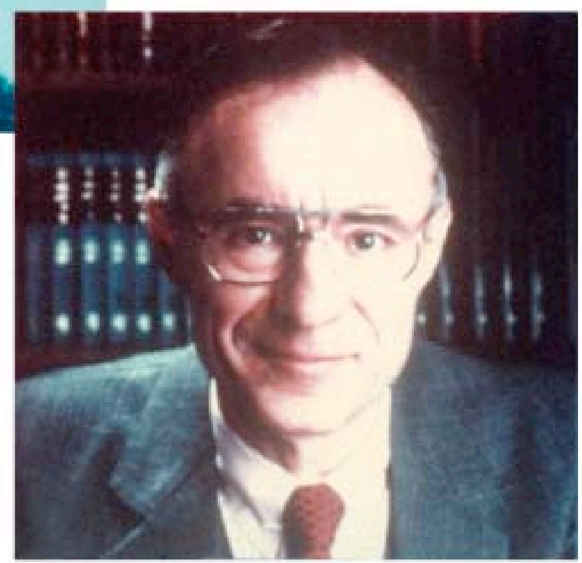
- Errors in both Velocity and Distance measurements - what is the best estimate of the Hubble constant??
- A Bayesian approach to this problem later…

Discovery of Cosmic Background



Microwave Receiver

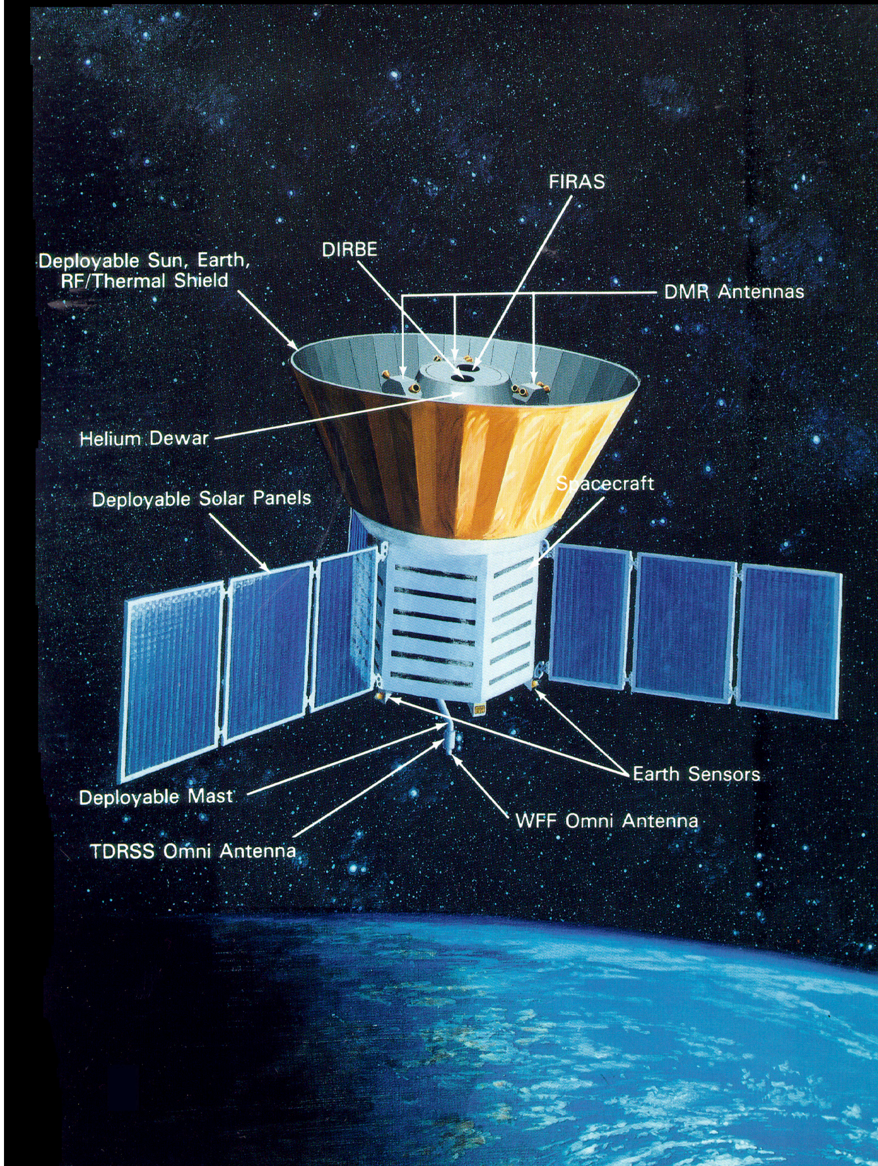
Robert Wilson

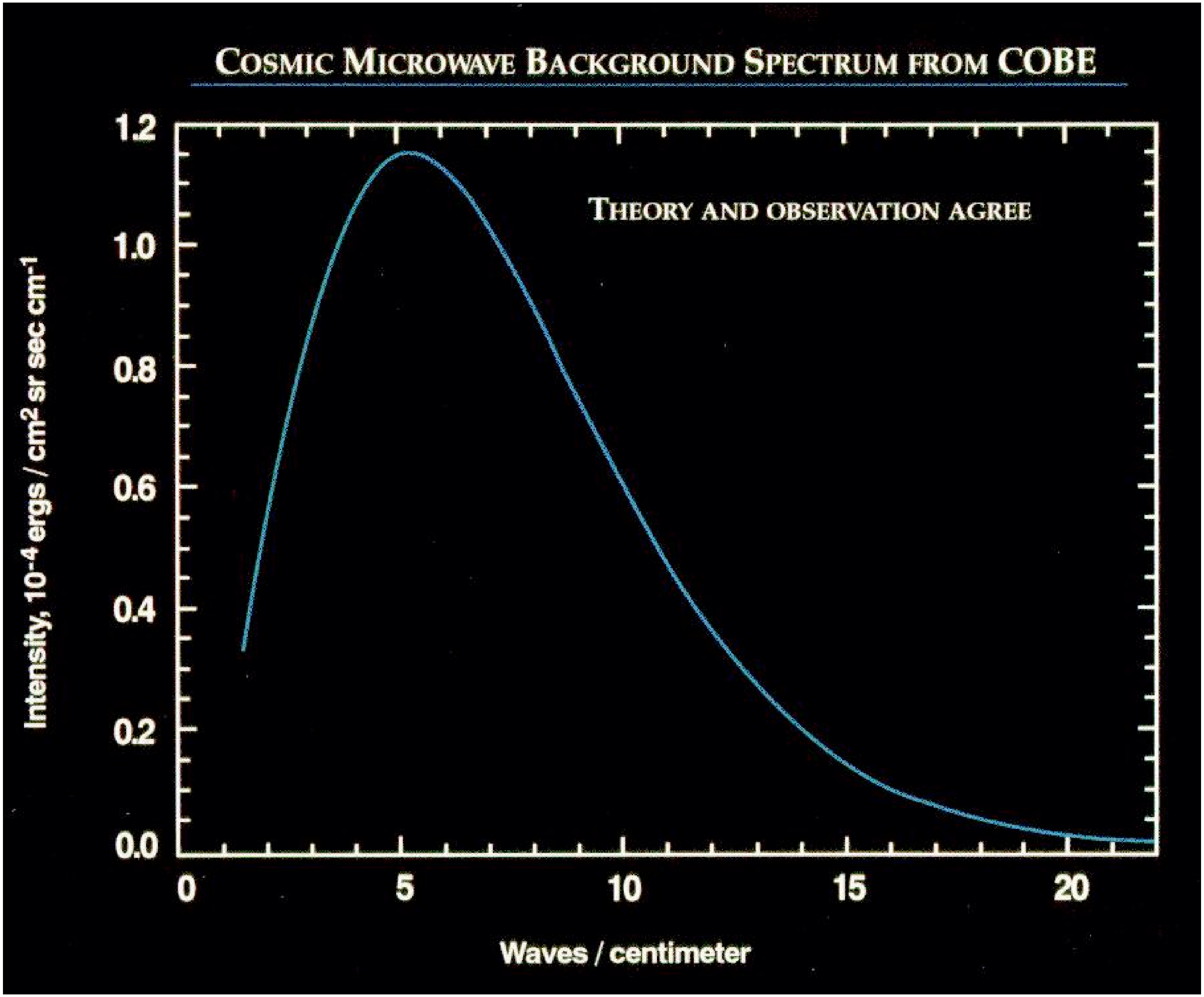


Arno Penzias

MAP990045

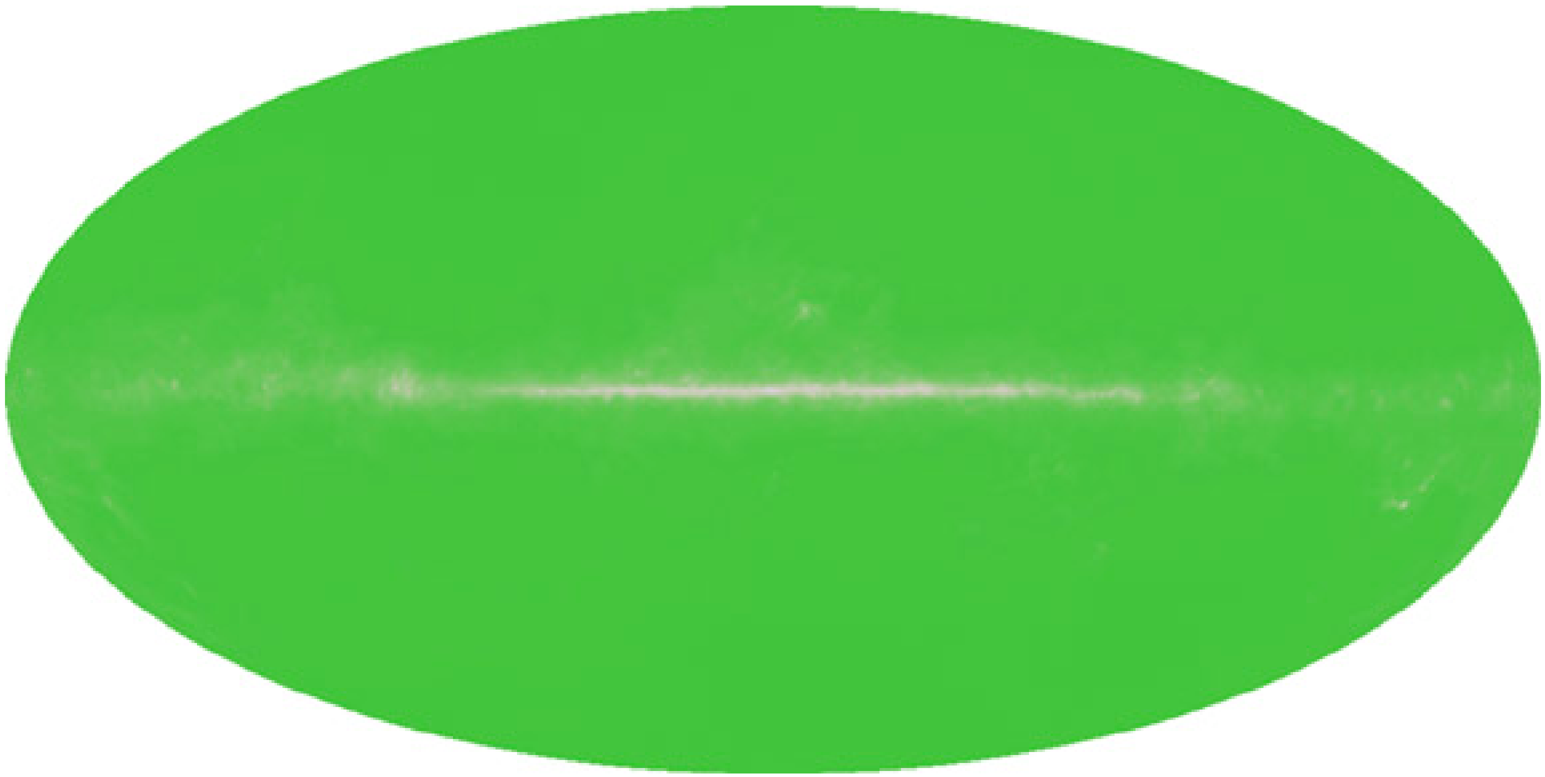
The COBE Satellite





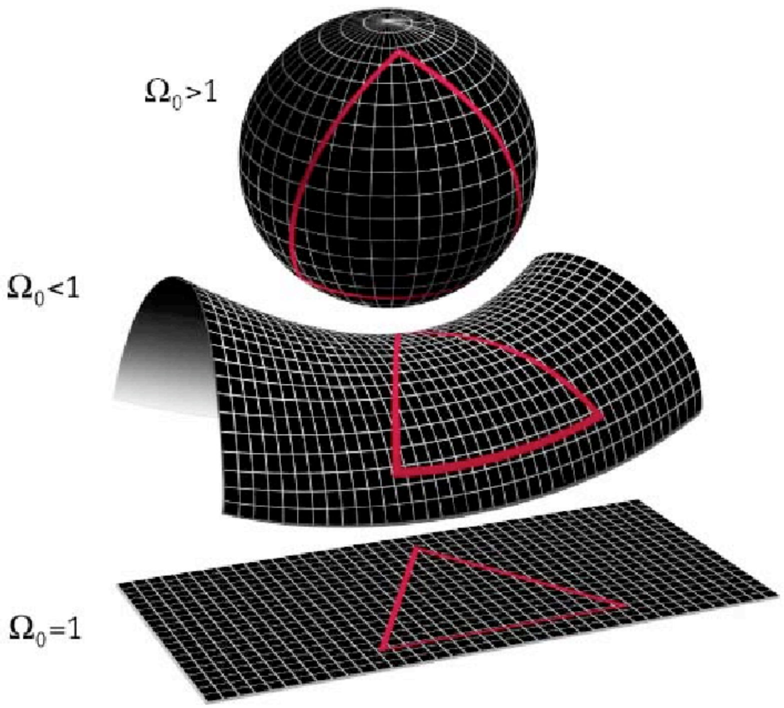
Cosmic Microwave Background Spectrum from COBE
Theory and observation agree
Intensity, 10^-4 ergs / cm^2 sr sec cm^-1
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Waves / centimeter

The Horizon Problem…



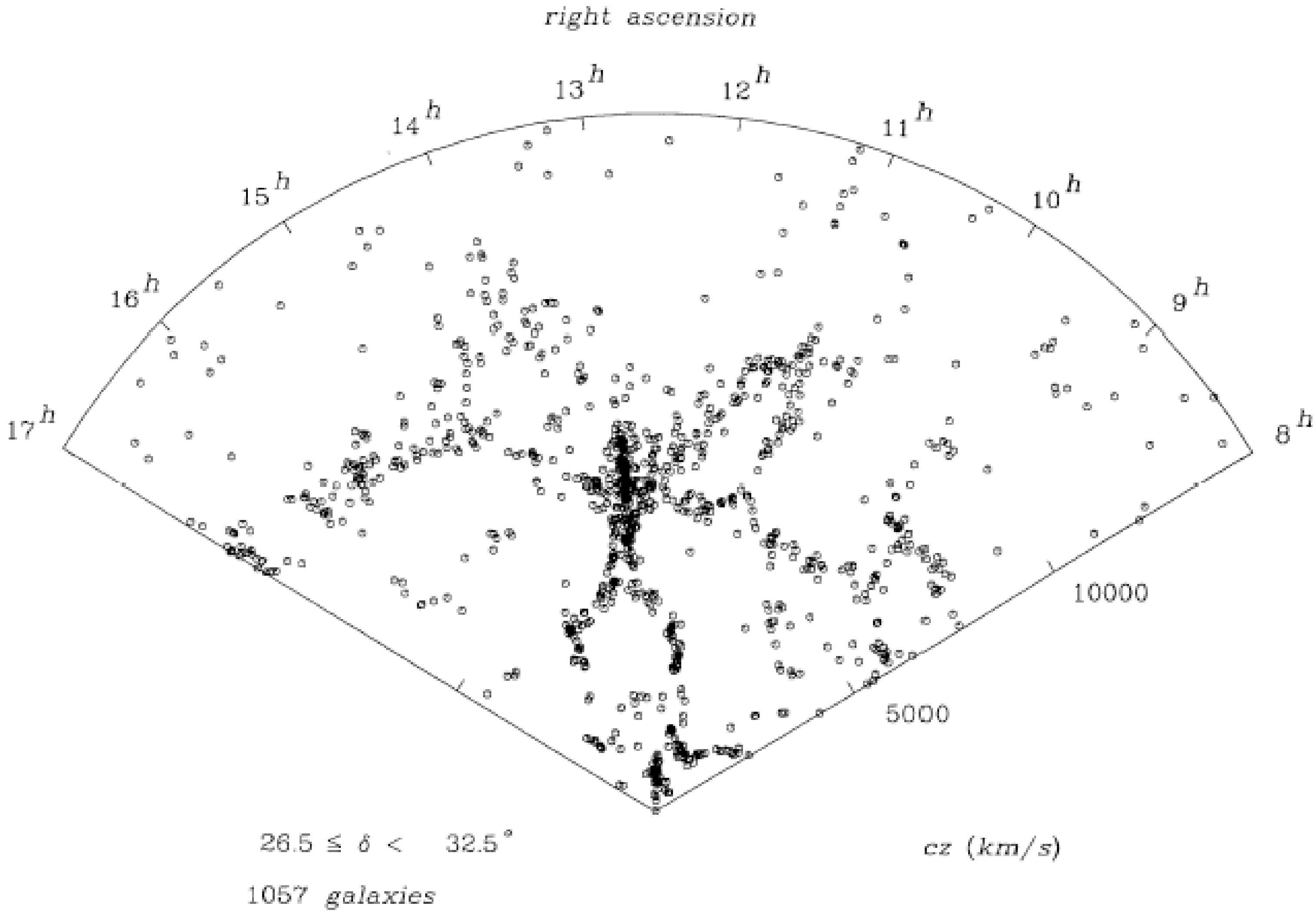
- Why is the CMB temperature so uniform across the sky??

The Flatness Problem…



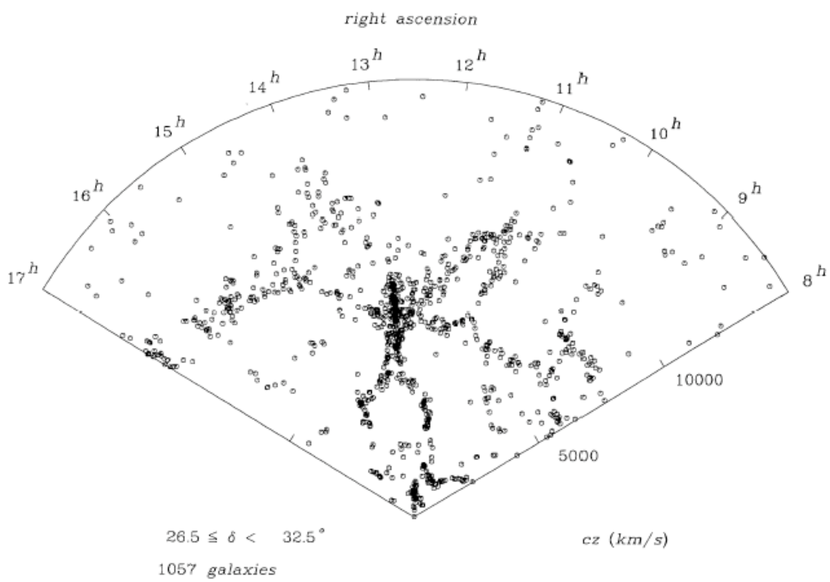
- Homogenous and Isotropic Universe can have three possible geometries
- Universe appears remarkably close to flat - why?

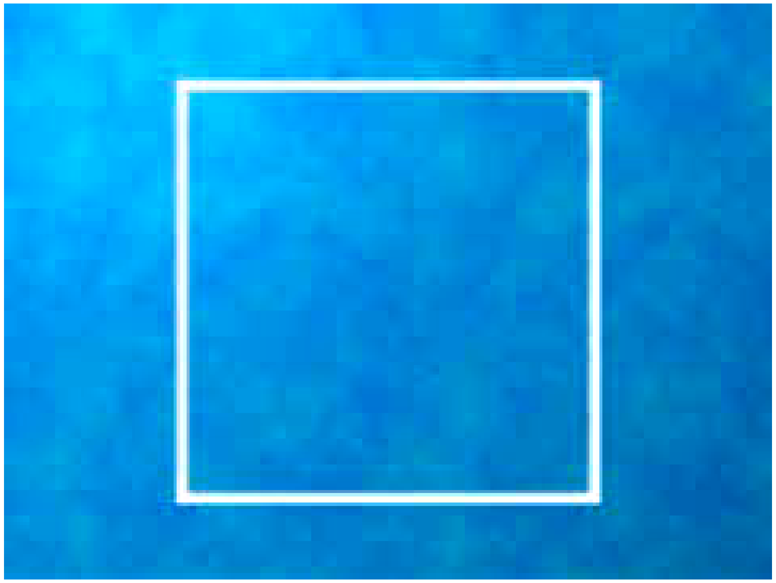
Early Redshift Surveys



Huchra et al, 1990ApJS...72..433H

Gravitational Collapse to form Large-Scale Structure

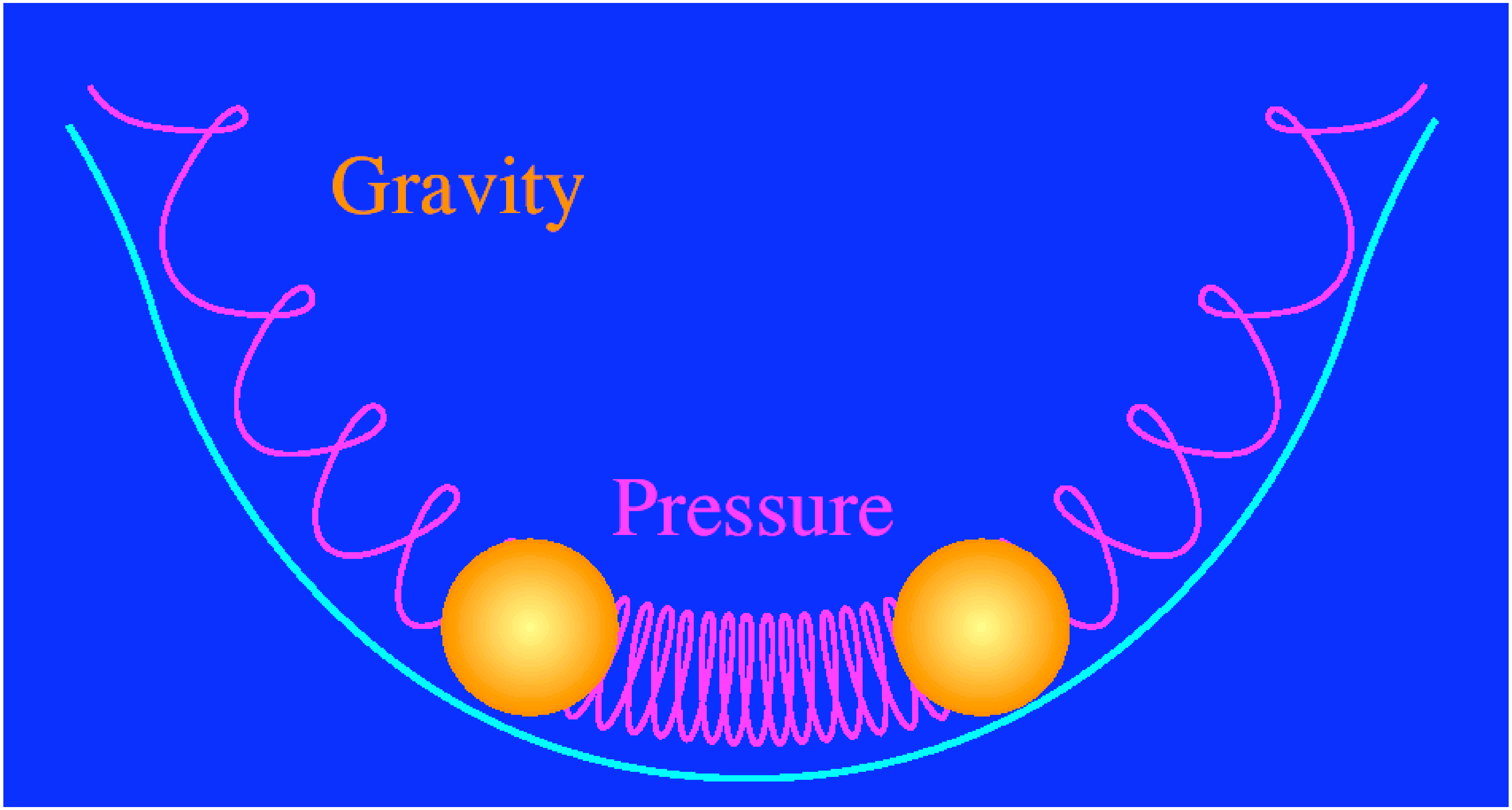




G. Bryan, and M. Norman, Lab for Computational Astrophysics, Univ. of Illinois, Urbana-Champaign

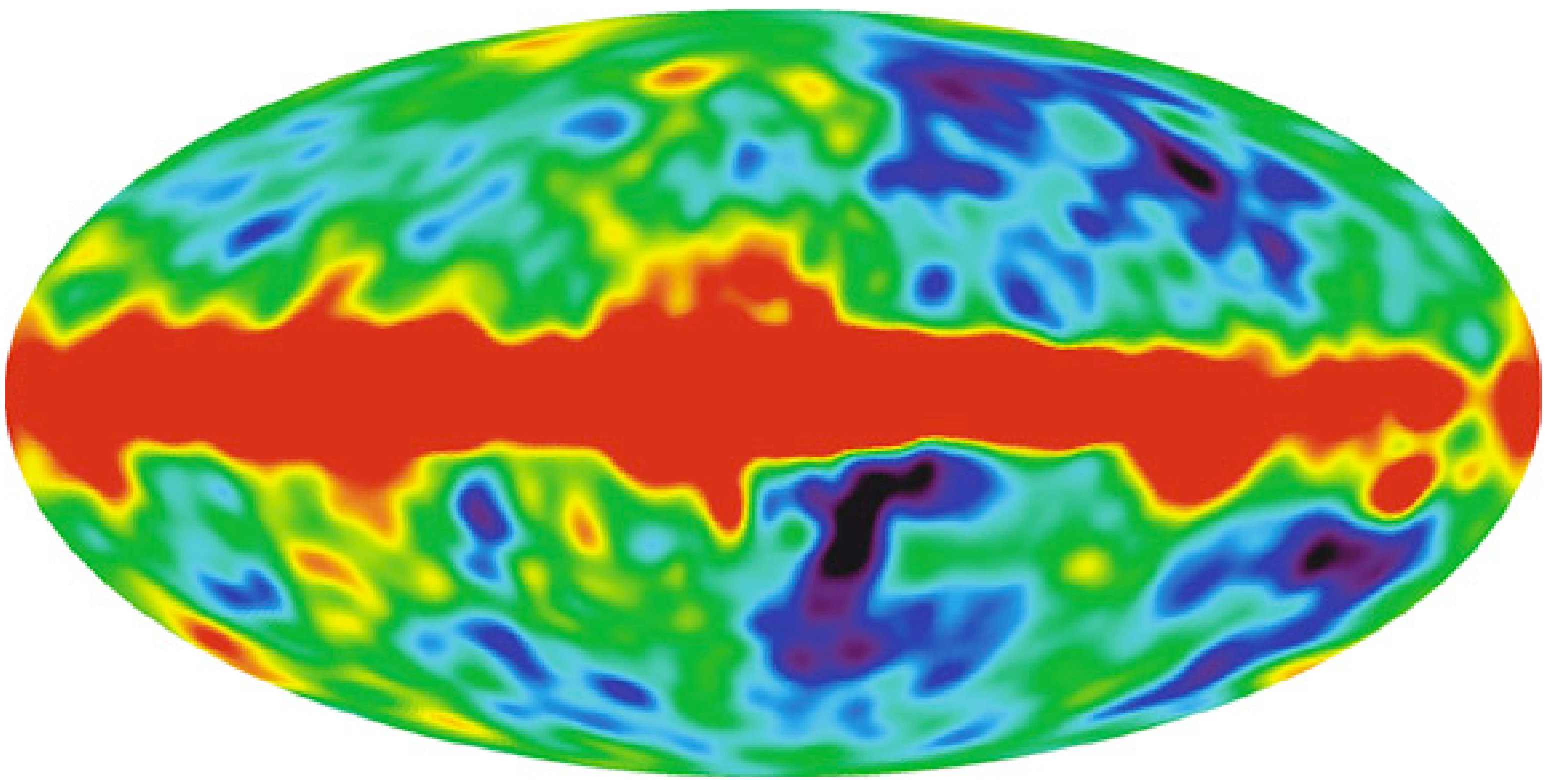
http://zeus.ncsa.uiuc.edu/mpeg/comoving.mpg

Fluctuations in the CMB

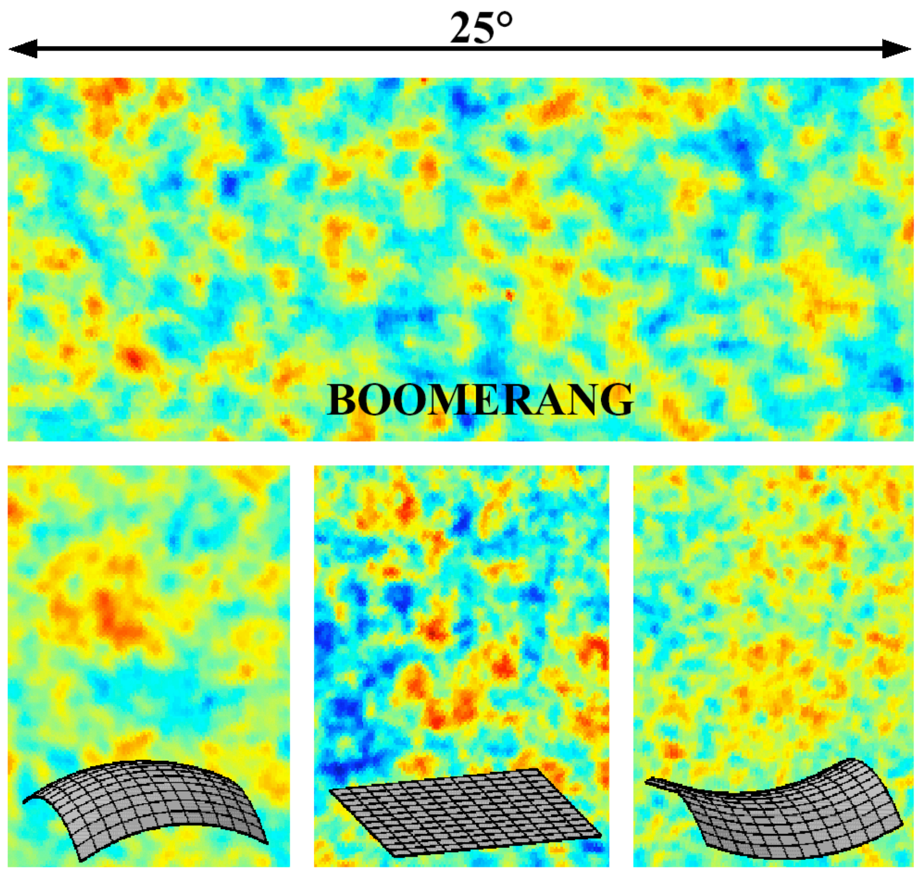


W. Hu, see http://background.uchicago.edu/~whu/

Seeds of Large-Scale Structure Detected!



Next Step: Inferring the Curvature of the Universe!



$$p(\square \mid d) \qquad p(d \mid \square)\, p(\square)$$

Example: Bayesian Inference of the Hubble Constant

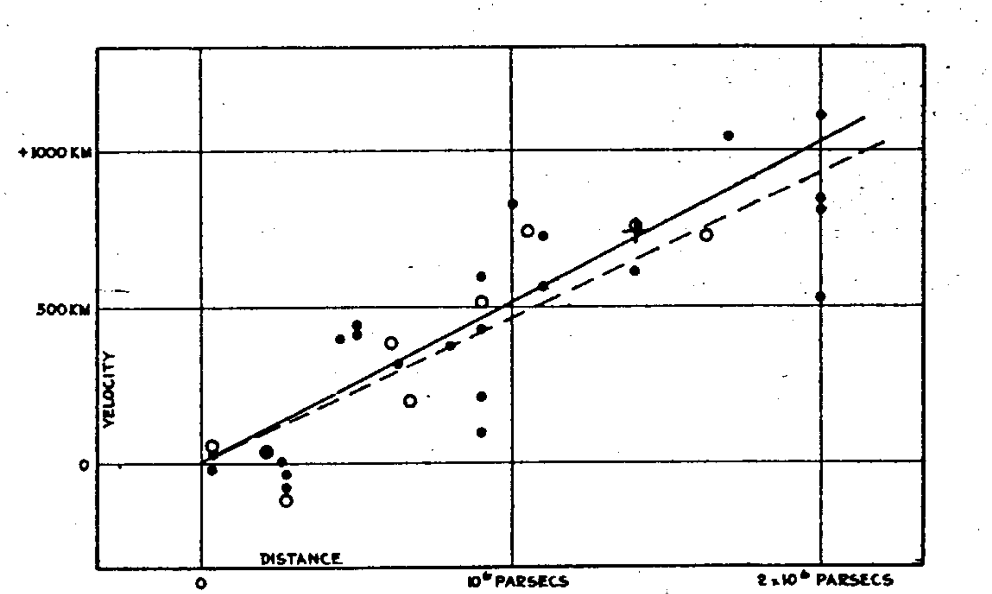


FIGURE 1

$$\boxed{v_i = H_0 r_i}$$

Only true "on average"

Bayes Theorem for the Hubble constant:

$$p(H_0 \mid v_i, r_i) \qquad p(v_i, r_i \mid H_0)\, p(H_0)$$

Bayesian Inference of the Hubble Constant

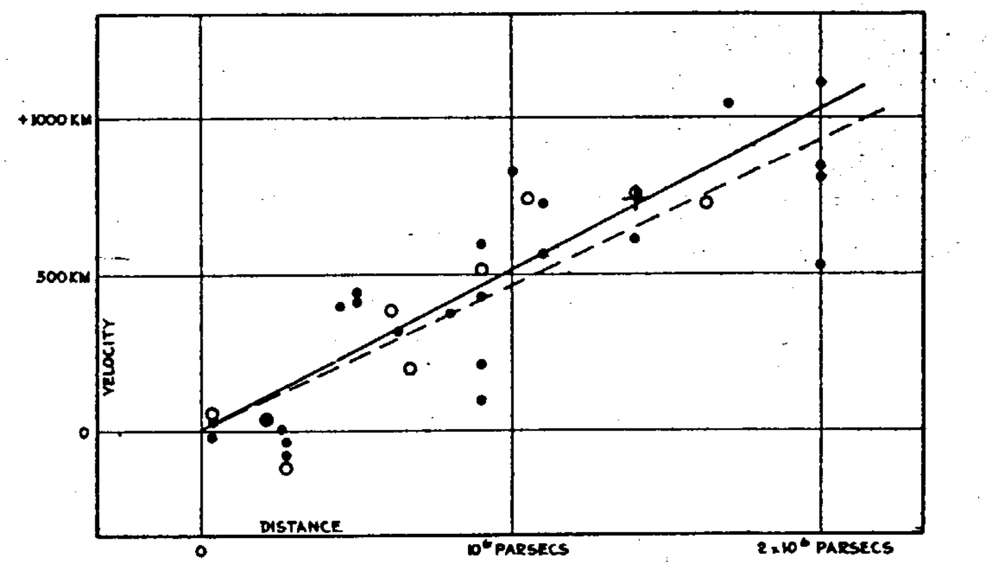


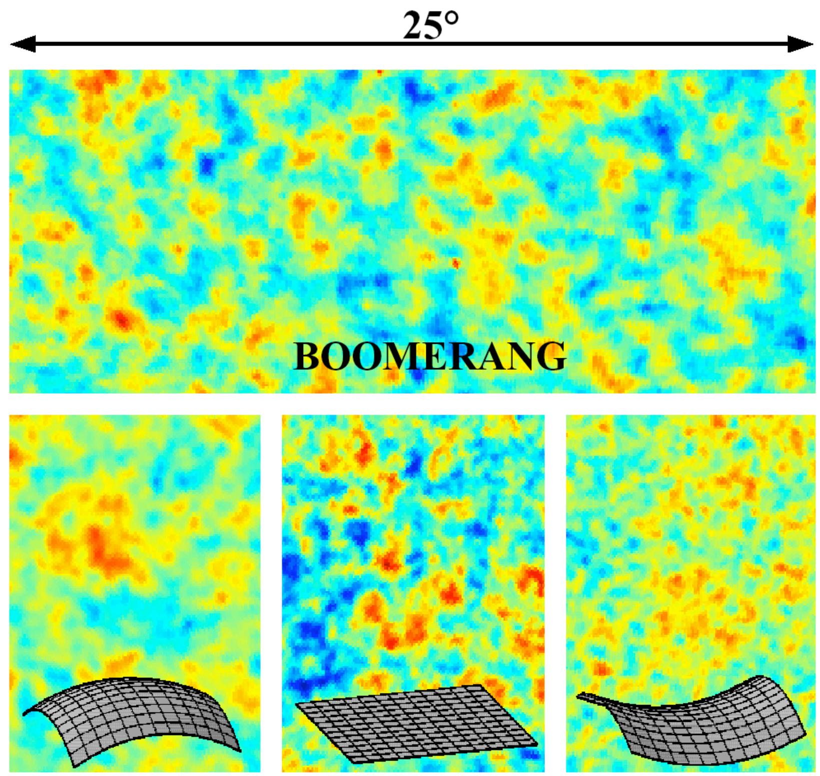
FIGURE 1

$$v_i = H_0 r_i$$

Only true “on average”

$$p(H_0 \mid v_i, r_i) \propto p(H_0) \frac{e^{-(v_i - H_0 r_i)^2 / 2(\sigma^2 + H_0^2 \sigma^2)}}{\sqrt{2\pi}(\sigma^2 + H_0^2 \sigma^2)^{1/2}}$$

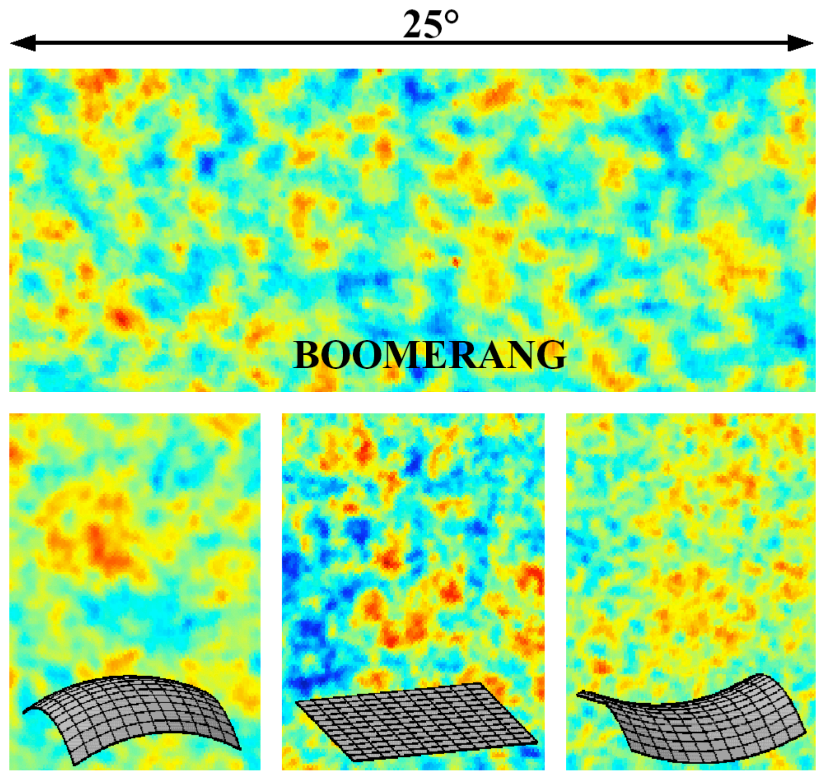
Inferring the Curvature of the Universe…



$$p(\Box \mid d)\,p(d) = p(d \mid \Box)\,p(\Box)$$

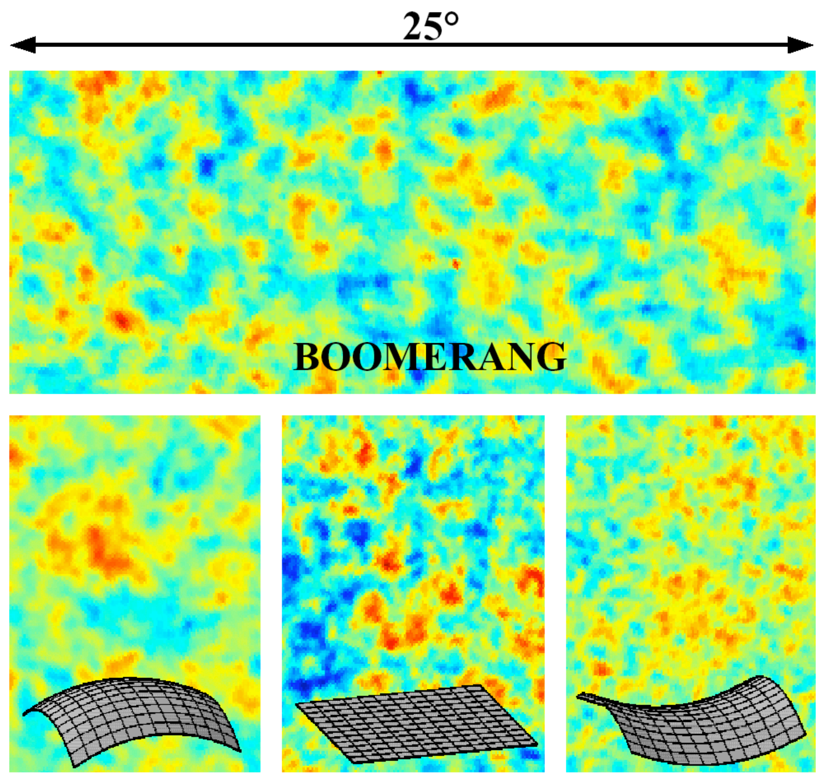
“Backward”=Inference　　　“Forward”=Simulation

No Noise: Simulation



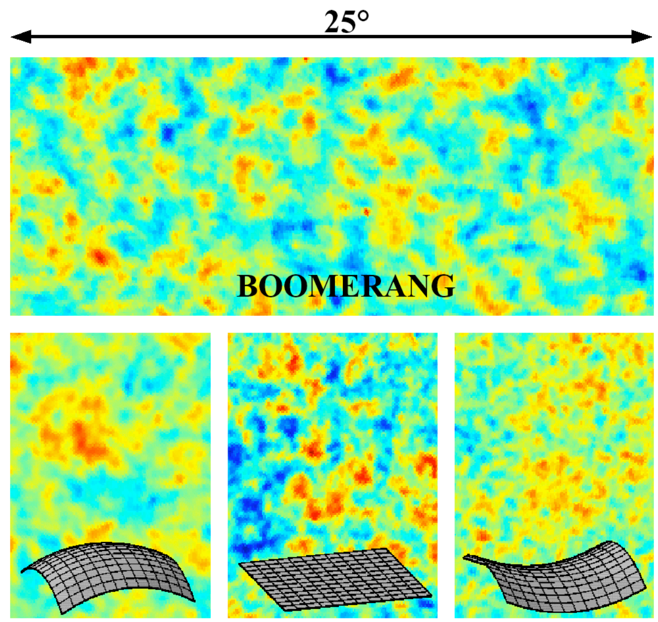
$$p(s \mid \Lambda) = \prod_{lm} \frac{e^{-\left|\langle lm \mid s \rangle\right|^2 / 2C_l(\Lambda)}}{\sqrt{2\pi C_l}}$$

No Noise: Inference



$$p(\theta \mid s) \propto p(\theta) \prod_{lm} \frac{e^{-\left|\langle lm|s\rangle\right|^2 / 2C_l(\theta)}}{\sqrt{2\pi C_l}}$$

Inference with Noise…



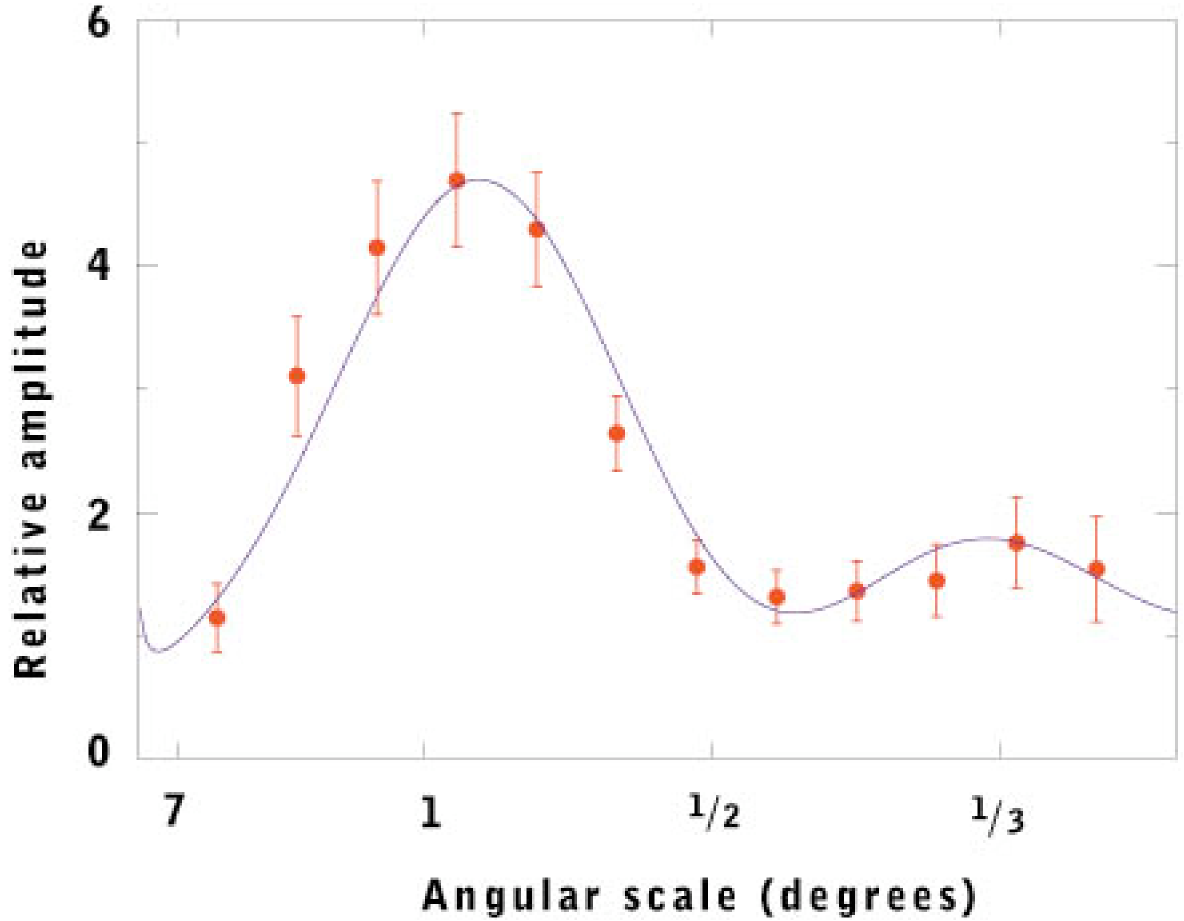
1) Write down the "forward" probabilities needed for *simulation*

$$p(\square, s, d) = p(d \mid s)\, p(s \mid \square)\, p(\square)$$

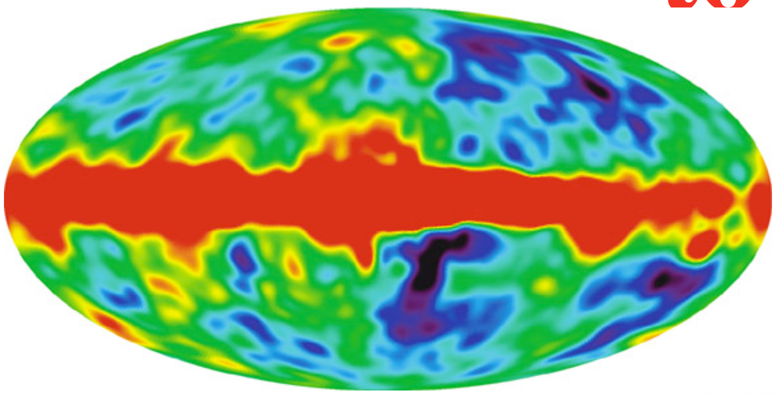
2) Solve for the Bayesian Posterior as a conditional probability

$$p(\square \mid d) = \int ds\; p(\square, s \mid d)$$

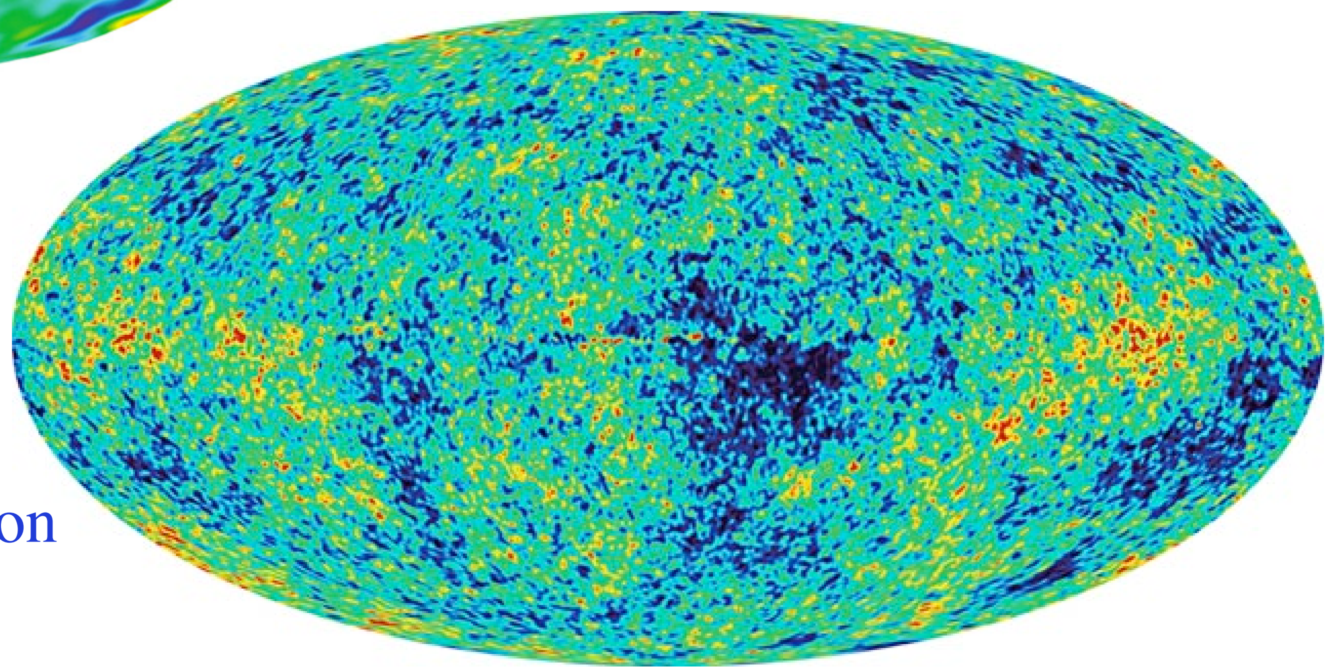
Power Spectrum - At One Degree Angular Resolution



Recent Results: From COBE to WMAP

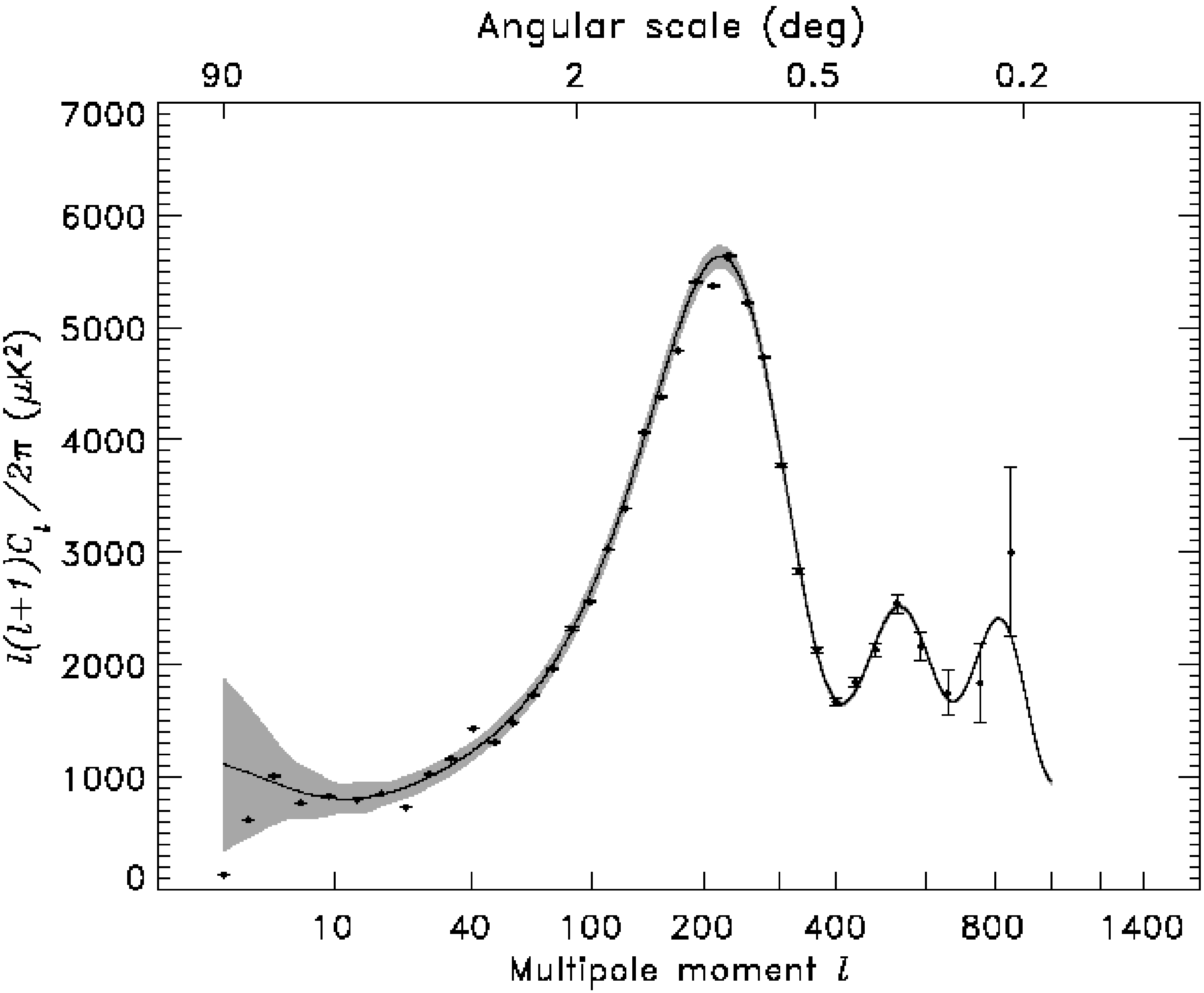


7 degree resolution



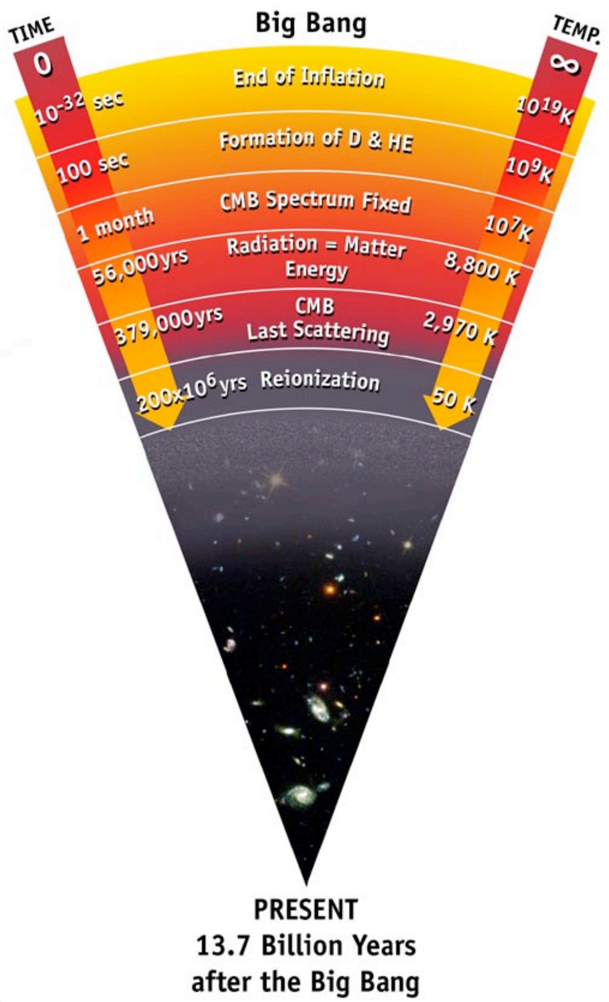
1/2 degree resolution

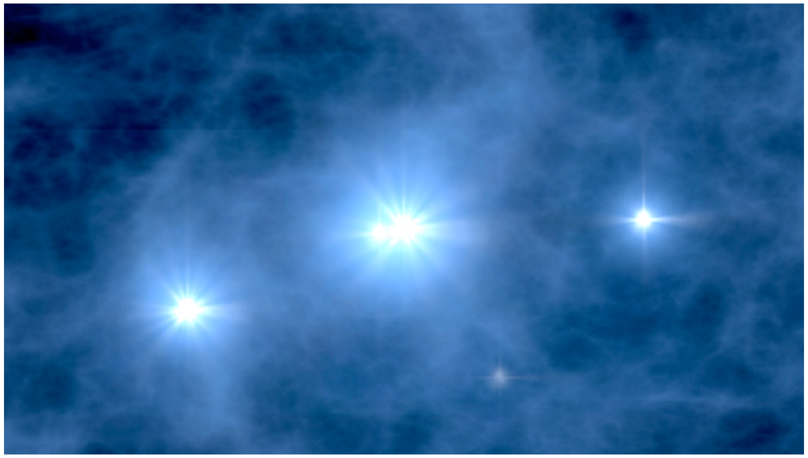
http://map.gsfc.nasa.gov

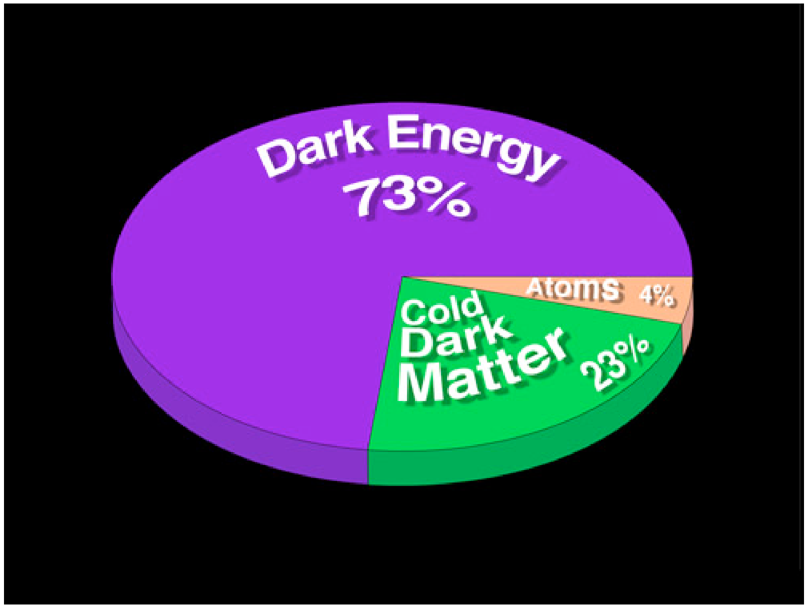


Angular scale (deg)
90
2
0.5
0.2
7000
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l\ /2\pi\ (\mu K^2)$
10
40
100
200
400
800
1400
Multipole moment l

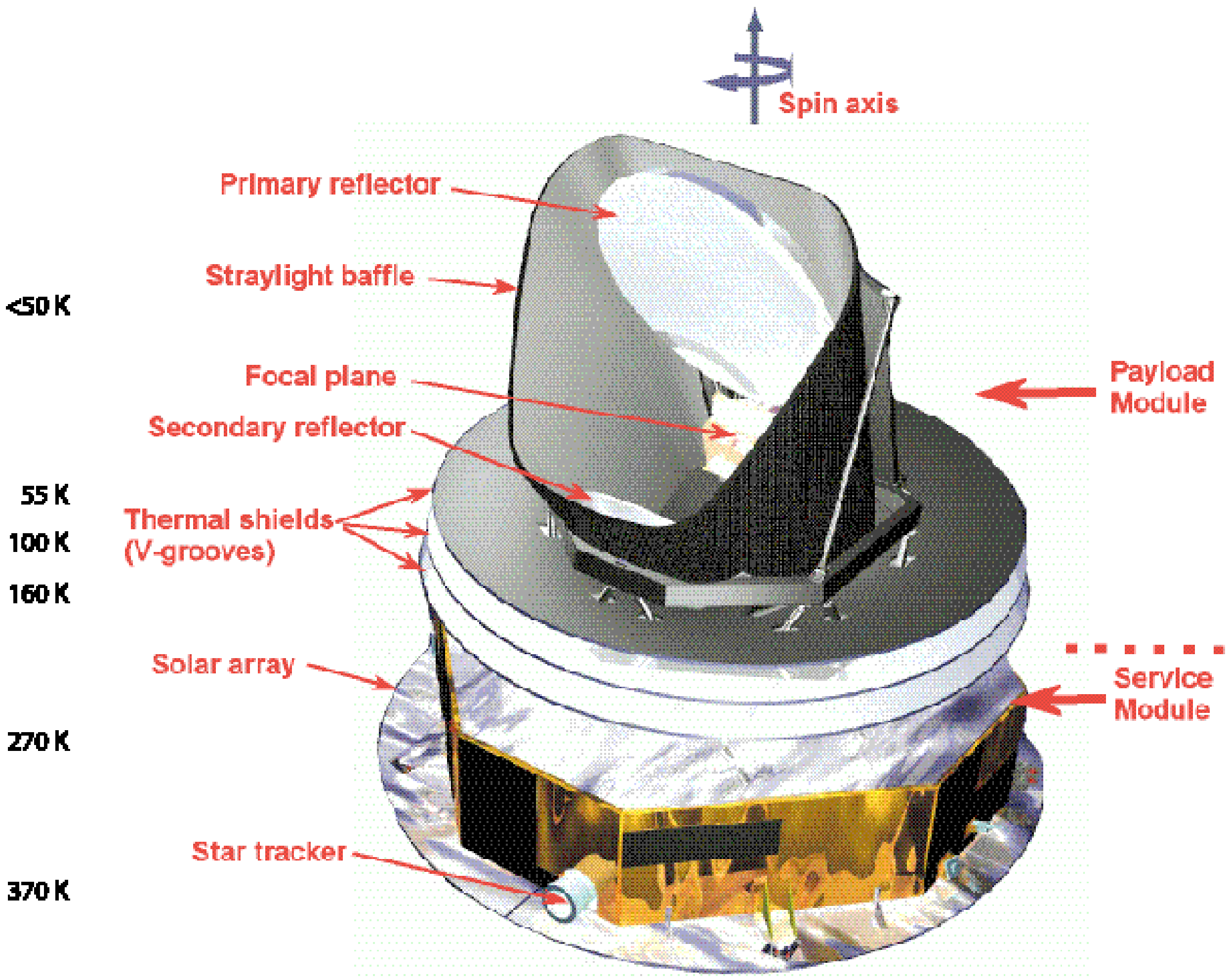
WMAP Results…

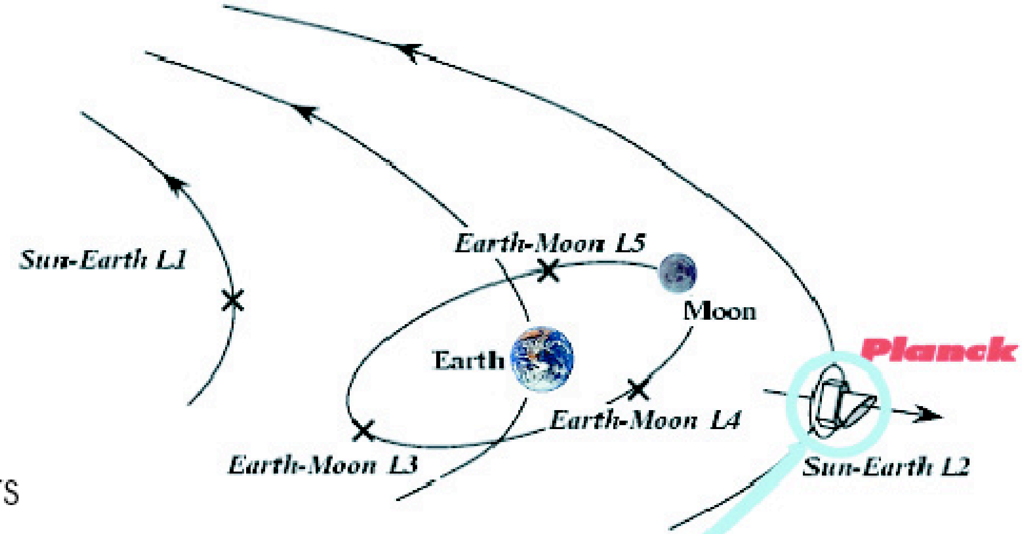




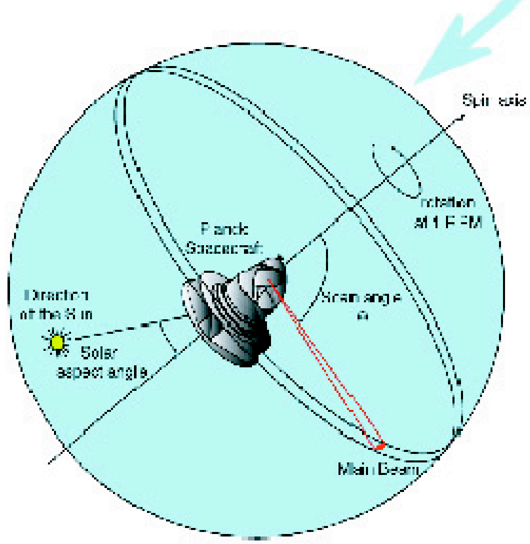


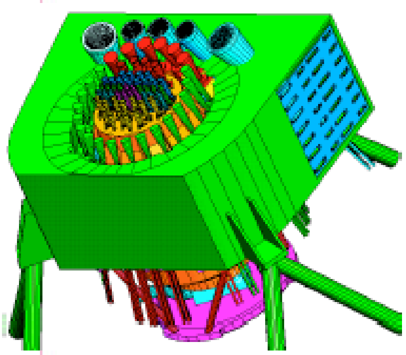
JPL's Role in the Future of CMB Studies - Planck



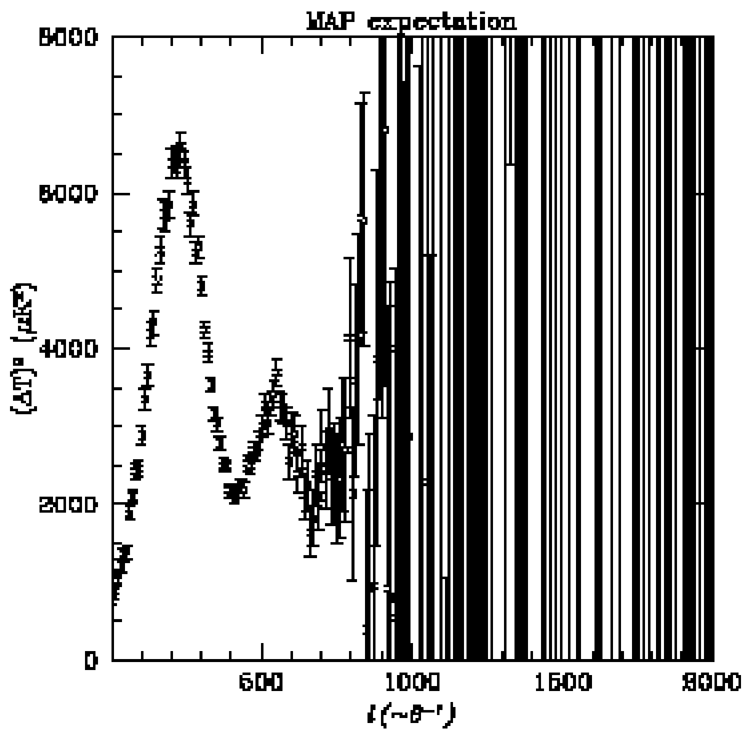


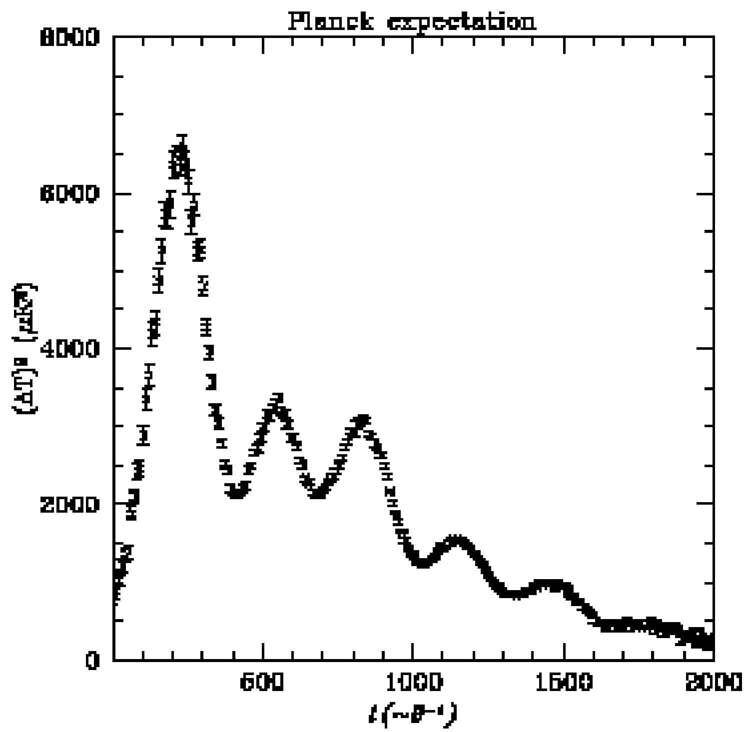
- Planck will spin at 1 rpm with its spin axis aligned with the Sun
  - Instruments scan nearly great circles on the sky
  - Entire sky observed every six months



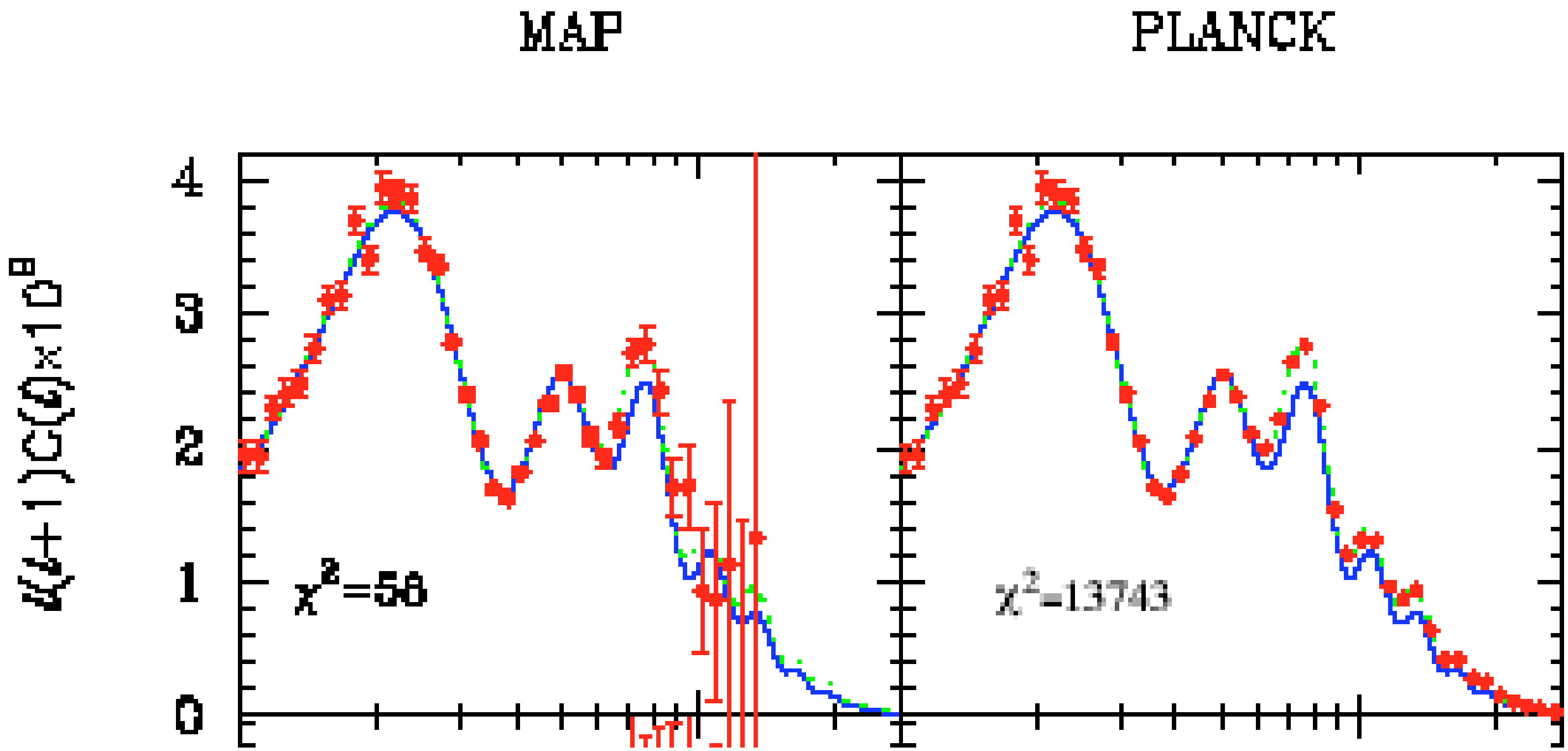


MAP vs. Planck - Expected 4 Year Sensitivity

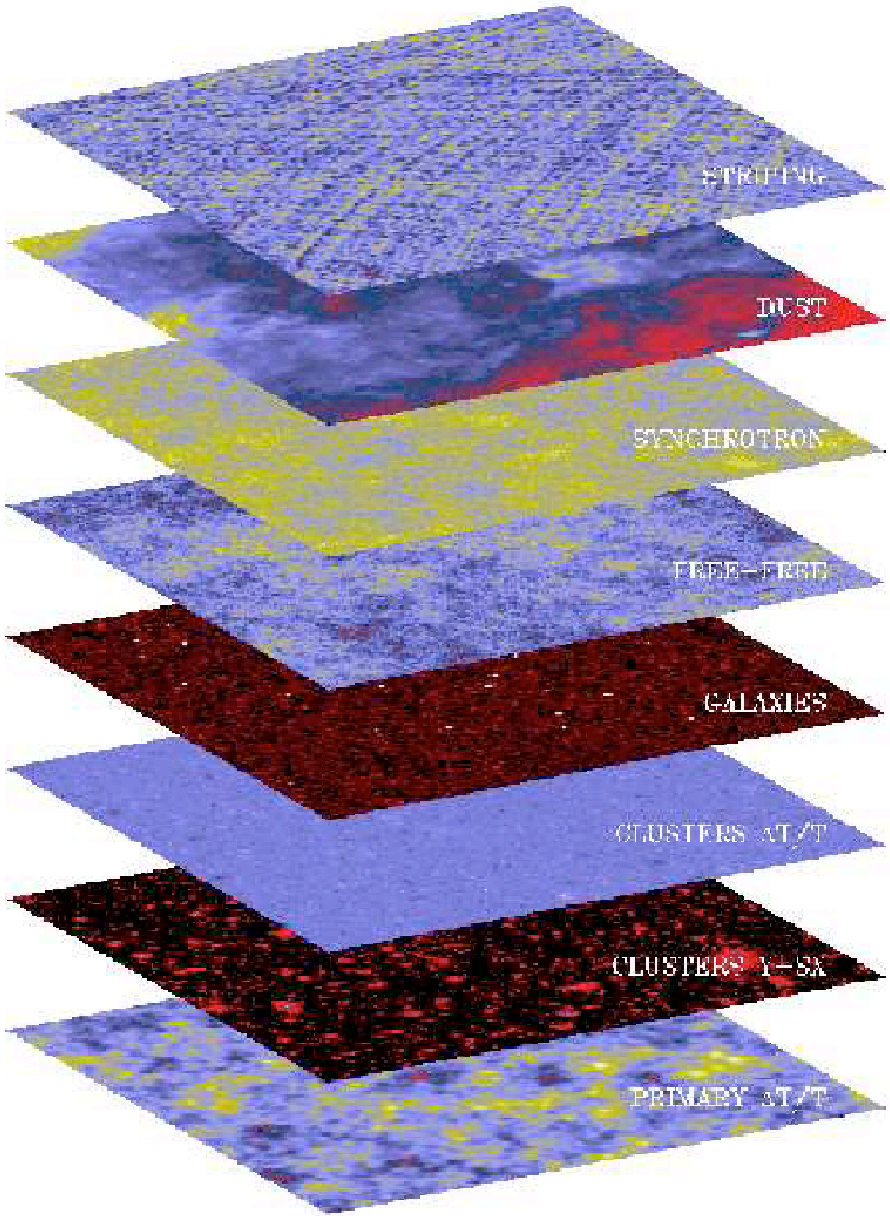




Planck: Resolving the Composition of the Universe!



- Solid line shows a spatially flat model
- Dotted line shows a model with 24% variation in baryonic density, and 5% variation in cold dark matter density…



Striping

Dust

Synchrotron

Free-Free

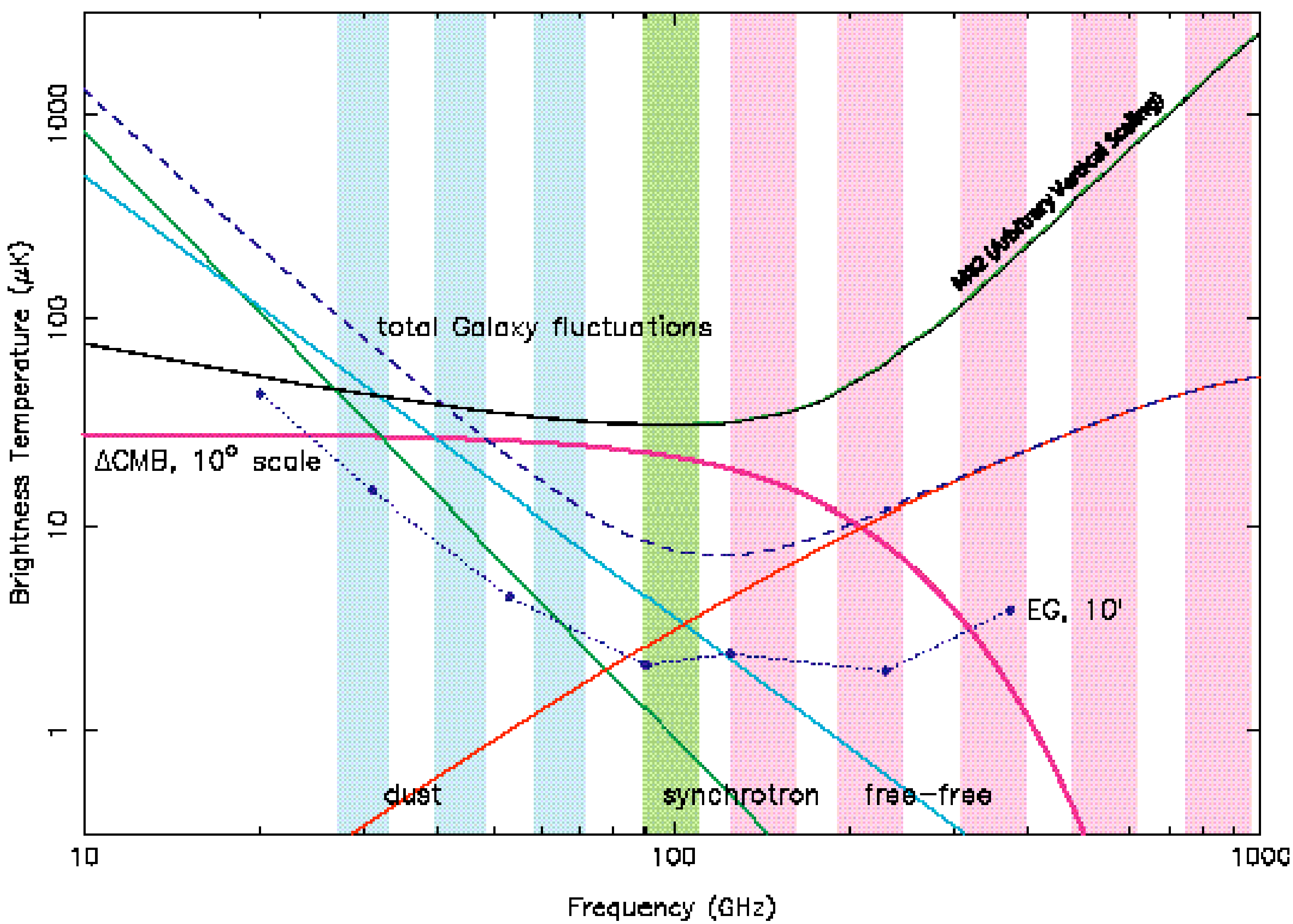
Galaxies

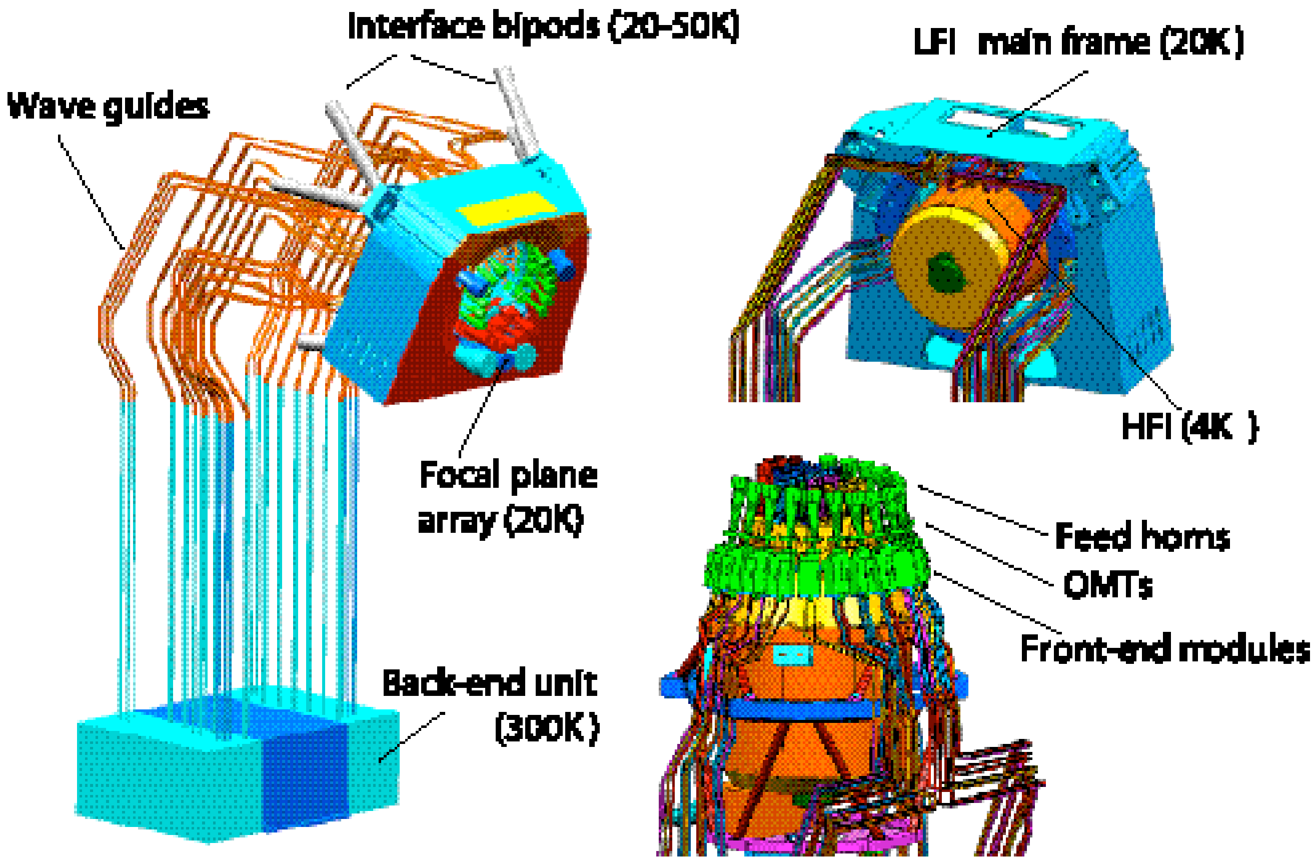
Clusters SZ Thermal

Clusters SZ Kinetic

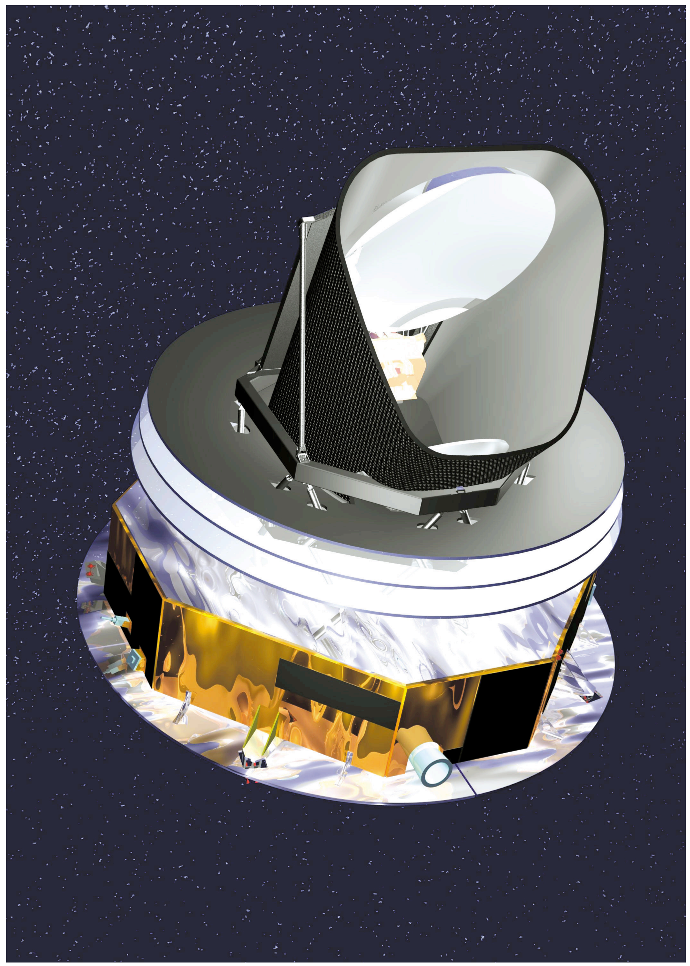
CMB

Foreground Frequency Response

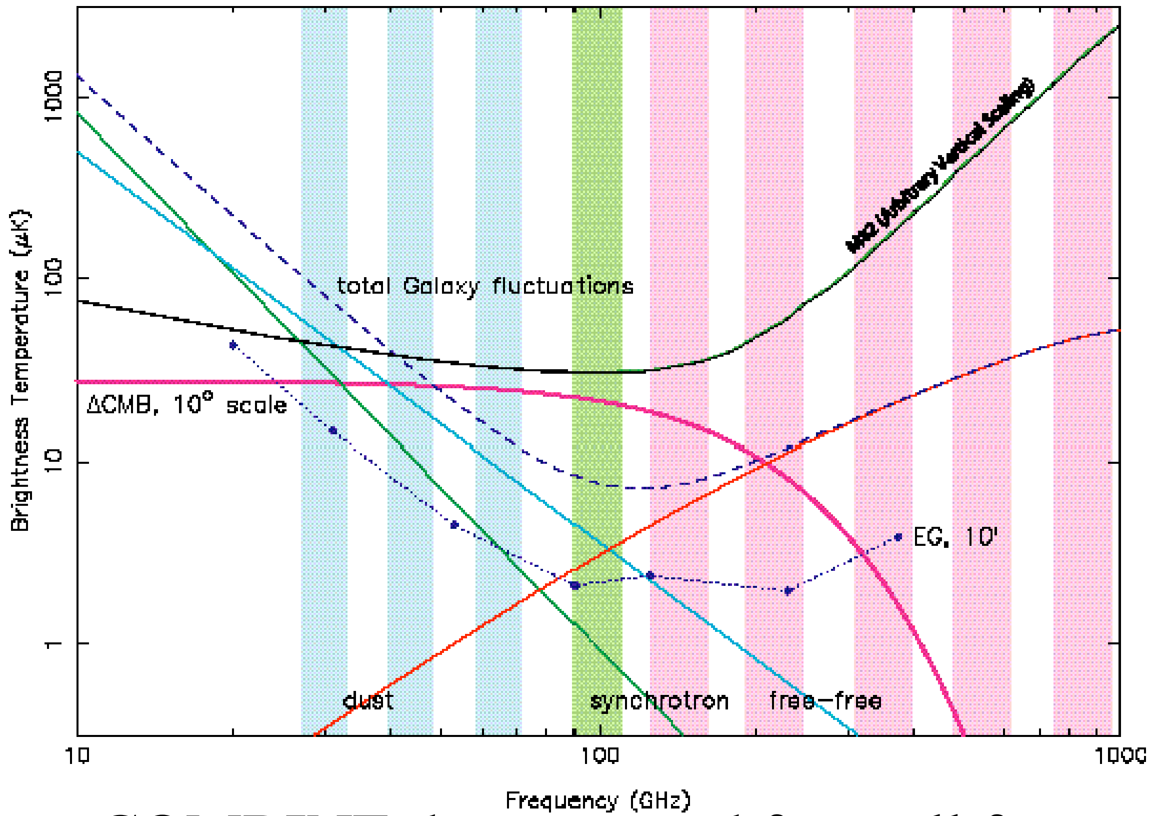



Interface bipods (20-50K)
LFI main frame (20K)
Wave guides
HFI (4K)
Focal plane
array (20K)
Feed horns
OMTs
Front-end modules
Back-end unit
(300K)

Planck -Computational Challenges



Combining Data Sets



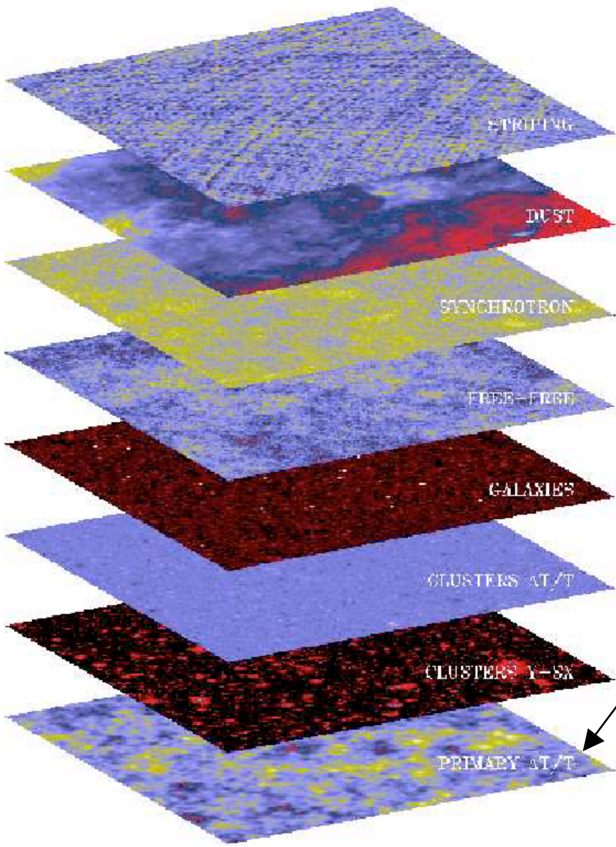
- Want to COMBINE data returned from all frequency channels, each with their own resolution, and noise properties
- Previous example: infer the Hubble constant from two very different types of measurements

Direct Maximization…

Computational Expense: $O[N^3]$

Flight	$\mathcal{N}_p$	Disc	RAM	Operations	Serial Time	Cray T3E Time
BOOMERanG NA	26,000	110 Gb	11 Gb	7.1×10^{14}	14 days	5 hours (64 PE)
MAXIMA 1	32,000	170 Gb	17 Gb	1.3×10^{15}	25 days	9 hours (64 PE)
MAXIMA 2	80,000	1 Tb	100 Gb	2.1×10^{16}	13 months	18 hours (512 PE)
BOOMERanG LDB	450,000	30 Tb	3 Tb	3.7×10^{18}	196 years	140 days (512 PE)

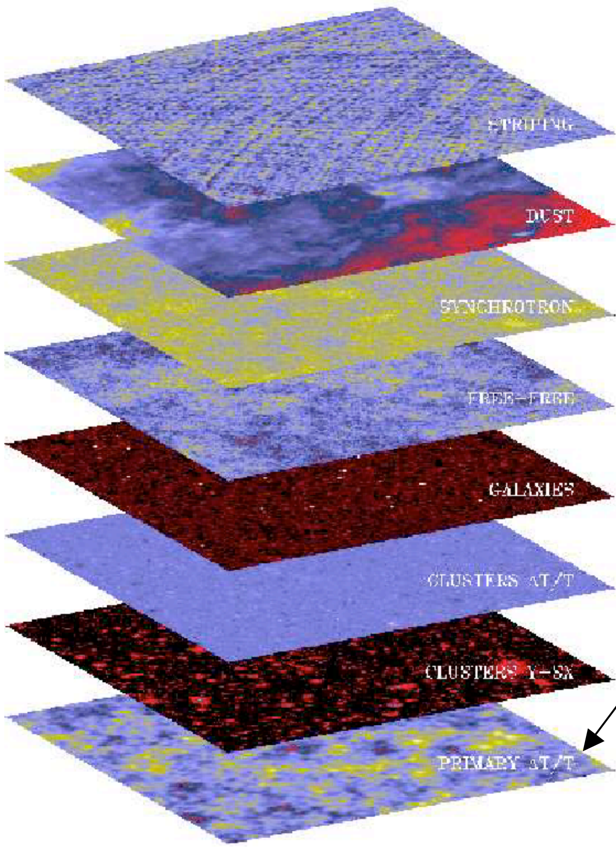
No Noise: Inference is Easy



Can we isolate the CMB and THEN estimate the power spectrum??

$$p(\theta \mid s) \propto p(\theta) \prod_{lm} \frac{e^{-\left|\langle lm \mid s\rangle\right|^2 / 2C_l(\theta)}}{\sqrt{2\pi C_l}}$$

Computing the Posterior…



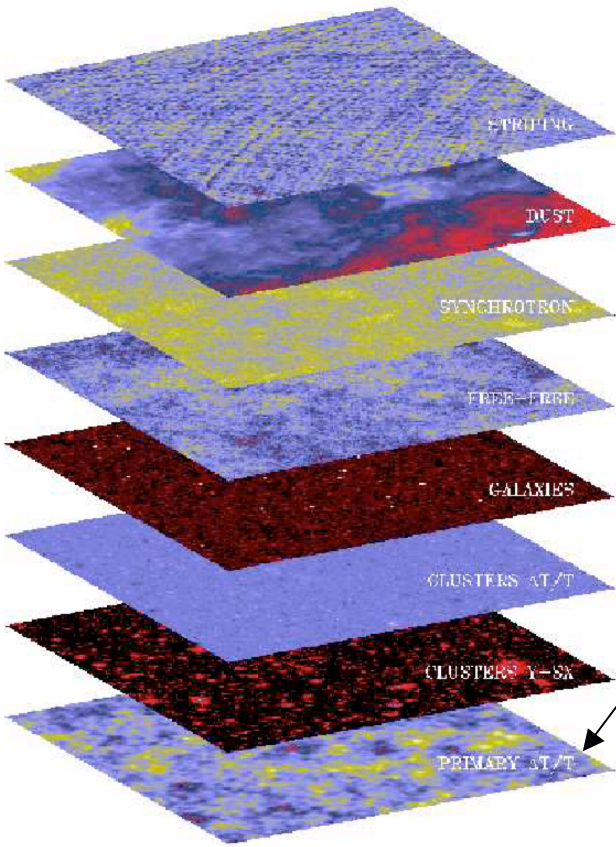
Can we isolate the CMB and THEN estimate the power spectrum??

No more expensive than forward simulation!!

$O(N^2)$

$$p_N(\Box \mid d) = \Box d \Box \Box \left[\Box ds\ p(\Box \mid s, d)\, p(s \mid \Box \Box, d) \right]^N g(\Box \Box\, d)$$

Computing the Posterior…



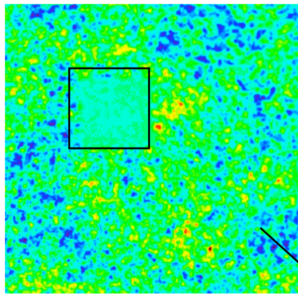
Can we isolate the CMB and THEN estimate the power spectrum??

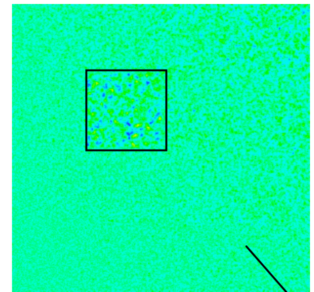
Conditionally independent of the data!!

$$p(\square \mid s, d) = p(\square \mid s)$$

$$p_N(\square \mid d) = \square d\square \square \left[\square ds\; p(\square \mid s)\, p(s \mid \square \square, d) \right]^N g(\square \square d)$$

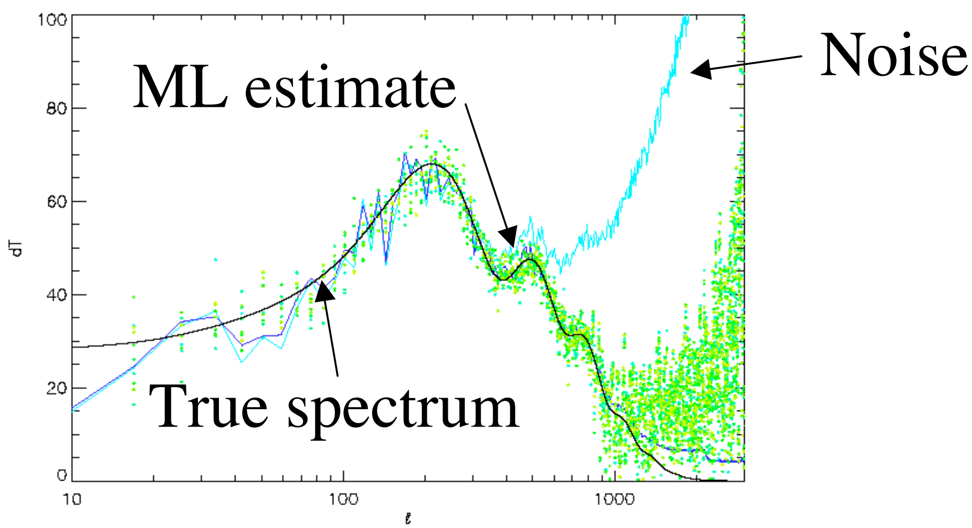
Monte Carlo Bayesian Power Spectrum Estimator





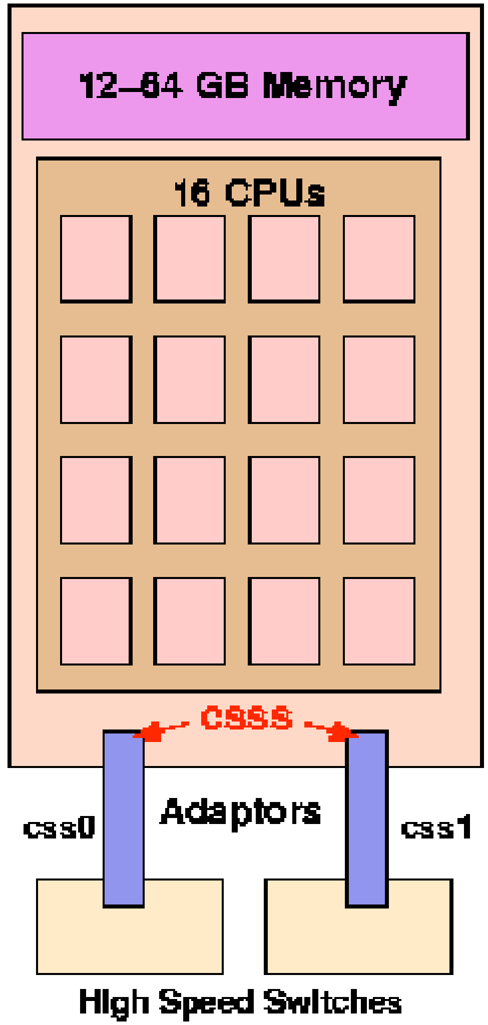
$$C_l(\theta_{n+1}) = \frac{1}{2l+1} \sum_{-l \le m \le l} \left\| \langle lm \mid \hat{s}(d,\theta_n) \rangle \right\|^2 + E\left[\left\| \langle lm \mid \xi \rangle \right\|^2 \mid d, \theta_n \right]$$

Iteration of Expectation Maximization algorithm gives result …

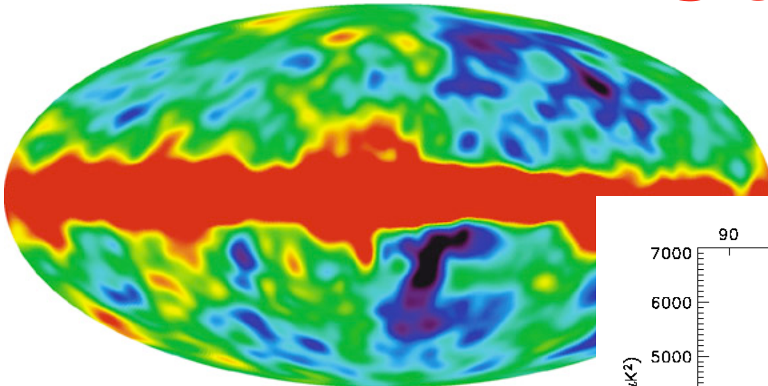


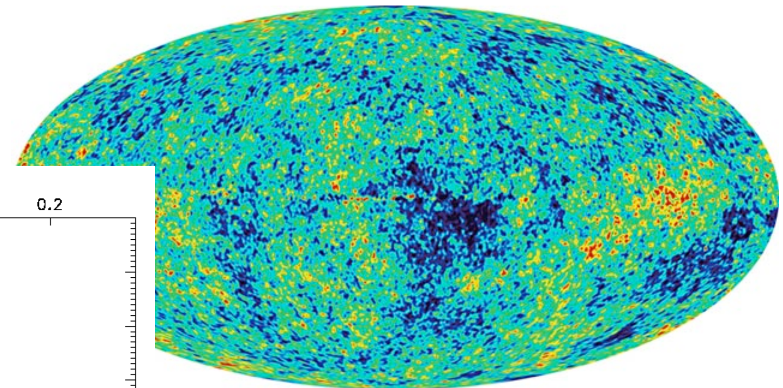
NERSC-Seaborg

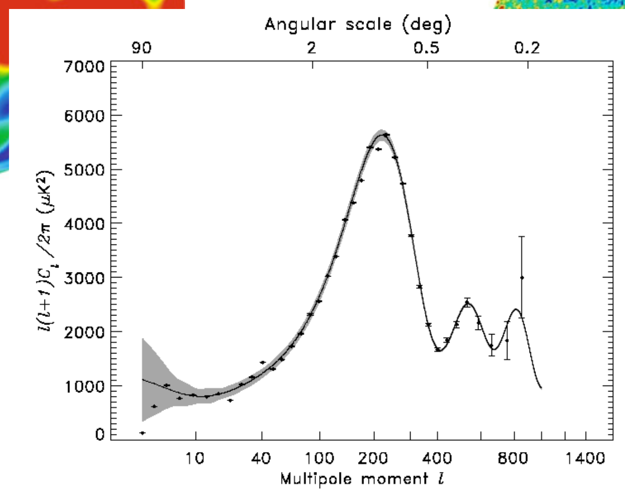
Over 400 nodes of 16 processors!!

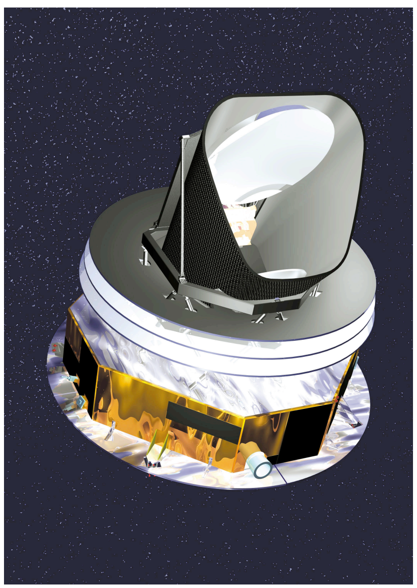


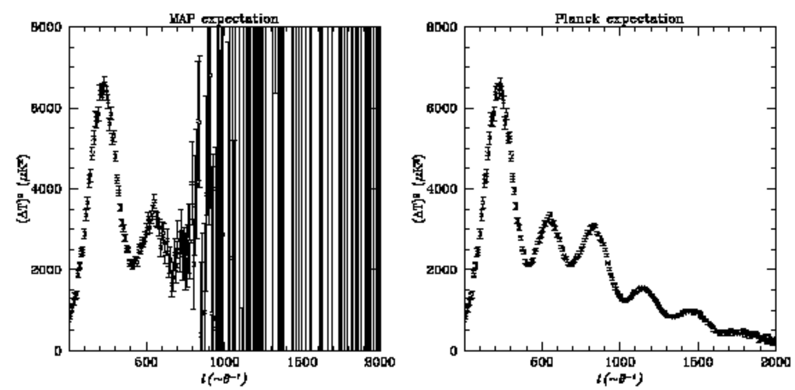
Conclusions











http://cs.jpl.nasa.gov/lectures.html